Industrial Capacity as a Military Deterrent*

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Abstract

Military outcomes depend not only on initial military strength but also on the capacity to expand it during war. We develop a model in which industrial capacity can have a strategic role as a military deterrent and use it to study how governments can sustain peace at the lowest cost. When wars end before wartime production can affect military outcomes, peacetime procurement suffices. When wars can be long, efficient deterrence also requires subsidies to industrial capacity because private investors do not internalize its strategic value as a military deterrent. Optimal policy gives larger subsidies to technologies that can expand output more readily during war. We develop an Integrated Defense Economics Model to quantify the gains from subsidies and identify priority sectors. Relative to a procurement-only benchmark, subsidies raise steady-state welfare and reduce military expenditure while maintaining deterrence. Priority sectors include guided missiles, propulsion units, and ammunition, and the model shows how these priorities vary with the type of conflict being deterred.

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1 Introduction

The war in Ukraine is an industrial war of attrition. In prolonged conflicts, munitions stocks can quickly be exhausted and equipment accumulated before the war destroyed. Wartime production can dwarf prewar inventories, making industrial capacity—and the ability to expand it—central to military success. The conflict has exposed shortfalls in Western production capacity and prompted renewed interest in industrial policy to prepare for prolonged war. The US, UK, EU and NATO have developed plans to strengthen their defense industrial bases, emphasizing both additional capacity and the ability to expand production rapidly.[1]

But the theoretical argument for such policies remains unclear. Why does the government need to support industrial capacity? Why can it not simply purchase military equipment and let firms choose their investments accordingly? What is the externality, and how does it depend on the duration of war? If support is warranted, which production technologies should it encourage? And how should it be targeted across sectors?

This paper studies optimal economic policy in a model of deterrence that places industrial capacity at the center. The military is one source of deterrence. But the defense industrial base can also contribute to deterrence by enabling military expansion during long wars. This creates a national security externality. The externality is absent if wars are expected to be short, and so purchasing military equipment suffices. But when wars are long, subsidies to industrial capacity are needed. These should aim to both expand the capital stock in the defense industrial base and shift its composition toward technologies that allow production to expand more readily during war.

We develop these qualitative insights in a simple baseline model in Sections 2 and 3. To answer our last question on how policy should be targeted across sectors, we need to operationalize the model. We need to model how equipment contributes to battlefield outcomes and how that contribution varies across stages of war. We therefore develop an Integrated Defense Economics Model (IDEM) that links frontline demand for military equipment to the industrial capacity needed to produce it. Section 4 introduces and calibrates IDEM, Section 5 presents the quantitative results, and Section 6 concludes with future directions for IDEM.

[1]The US National Defense Industrial Strategy calls for incentives to build spare capacity and develop flexible production (U.S. Department of Defense, 2023). The United Kingdom's Defence Industrial Strategy calls for maintaining flexible production capacity and assessing scalability in major procurements (Ministry of Defence, 2025). The European Defence Industrial Strategy proposes funding spare industrial capacity to enable rapid expansion during crises (European Commission and High Representative of the Union for Foreign Affairs and Security Policy, 2024). NATO's Updated Defence Production Action Plan likewise calls for industrial planning that addresses long-term scalability and surge requirements (NATO, 2025).

Baseline Model Our model of military deterrence places industrial capacity at the center. Consider a country that faces an adversary who contemplates starting a war to challenge the status quo through military force. Such a war can either be long or short, and for simplicity we assume both parties know how long it will last. During a war, territory changes hands depending on the relative strength of the militaries. During a short war, the military strength is determined by what a country starts with. During long wars, it also includes what it produces using its defense industrial base. To sustain the peace, the country needs to make war sufficiently costly for its adversary. It can do so during peace by investing in both the size of its armed forces and the size of the defense industrial base.

The key outcome is that industrial capacity can act as a form of deterrence. Deterrence does not just depend on the military equipment a country has available at the start of the war but also on what it can produce over the duration of a longer war. An adversary understands that a country with larger industrial capacity will leverage it to expand military capacity throughout the war. In a way, the peacetime defense budget determines the army a country starts with; industrial capacity determines what it can end up with. Because it affects an adversary's calculus of aggression, industrial capacity is not just a supplier to the military but can directly contribute to deterrence.

We study an equilibrium in which the government sustains the peace at all times. How can it do so at the lowest cost? We study two types of peacetime policy: *procurement* and *industrial* policy. Procurement consists of purchases of military equipment such as ammunition and tanks at market prices. Industrial policy influences the inputs firms use to produce that equipment. Since private firms choose investment in the defense industrial base, we focus on subsidies through which the government can encourage capital accumulation across sectors.[2]

Economists have traditionally distinguished sharply between these two types of policy. The classical policy prescription to solve the underprovision of public goods is to replace private provision with public provision—through government procurement of military equipment in our case. Economists have often been skeptical of additional interventions such as industrial policies beyond such procurement policies. The seminal production efficiency result of Diamond and Mirrlees (1971) justifies this skepticism. It shows that industrial policy, as defined above, is indeed not needed for efficient public good provision in standard neoclassical models in which public goods are provided by the government by combining goods with capital or labor.

[2]We highlight that this is a theoretical distinction between policy instruments. In practice, procurement contracts can combine what we call procurement and industrial policy by paying producers both for equipment and to retain additional production capacity.

Results Our first result provides a useful baseline: when wars are short, efficient deterrence only requires procurement policy. Short wars are defined as wars that end before wartime production can affect military outcomes, and so the defense industrial base has no direct role in deterrence. The defense industrial base acts as a producer of military equipment, but this by itself is not a reason to intervene because market prices lead to efficient investment in capacity. When wars are short, the assumption that national security is about the provision of deterrence therefore does not break with the production efficiency result of Diamond and Mirrlees (1971).

When wars are long, this is no longer the case. Our second result shows that efficient deterrence also requires industrial policy. By procuring military equipment at market prices, the planner rewards the private sector for minimizing the cost of equipment during peacetime. But policy should aim to minimize the cost of deterrence. During short wars, these problems coincide, but during long wars, they do not. The issue is that the government cannot procure deterrence directly. It only procures military equipment directly, but industrial capacity is chosen by the private sector.

Industrial capacity contributes to deterrence by enabling wartime production, allowing the government to sustain a given level of deterrence with smaller armed forces. But firms selling military equipment at market prices are not rewarded for their capacity to produce in a war that never materializes. The resulting national security externality leads to underinvestment in industrial capacity and justifies direct government support. Without such industrial policy, firms choose a production structure suited to relatively stable peacetime procurement, which need not be well suited to expanding output during war.

Our third result shows that industrial policy should favor more scalable technologies through larger subsidies. We extend the baseline to include two production technologies that differ in how readily output can expand with capital fixed. A technology is more scalable when output responds strongly to increases in flexible inputs and those inputs are supplied elastically. Together, these features allow production to expand without a sharp rise in marginal costs. Reallocating capital toward the more scalable technology can strengthen deterrence by increasing wartime production, even if the aggregate capital stock in the defense industrial base remains fixed.

The interaction between the two key model ingredients—deterrence as a public good and war duration—is central to these results. The former introduces a national security externality that can overturn the production-efficiency conclusion of Diamond and Mirrlees (1971) and create a role for industrial policy; the latter determines whether industrial capacity actually contributes to deterrence.

Quantitative Exercise In the final part of our paper, we study how industrial policy should be targeted across sectors. The reason why industrial capacity contributes to deterrence is that it results in more military equipment over the duration of the war, which improves battlefield outcomes. We therefore develop an Integrated Defense Economics Model (IDEM) to study how industrial capacity contributes to deterrence and how policy should be targeted. The IDEM combines a dynamic general equilibrium economic block with a model of military conflict.

The economic block models how industrial capacity contributes to the capacity to scale military output during a war. We use an input-output model that captures military producers' reliance on other industries for intermediate inputs. This lets the model capture the capacity to scale military output within a given period. We additionally model the capacity to scale military output over time by incorporating the production of capital goods through the investment network. This allows us to capture how an economy scales over time by expanding production of inputs that are sticky within a given period. We calibrate the model using U.S. production and investment networks.

The model of military conflict captures the contribution of military equipment to battlefield outcomes. The military combines various inputs to produce military strength. These inputs are equipment to be expended or destroyed at different rates. A country may go through its stockpile of missiles faster than its stockpile of larger equipment such as aircraft. We allow the expected duration of war to be uncertain and to depend on the initial balance of military power. We calibrate the model to match historical evidence on battlefield attrition, the duration of wars, and the size of military buildups during wars.

We find that industrial policy raises steady-state welfare relative to our procurement-only benchmark. It raises consumption by reducing military expenditure and raising the aggregate capital stock, holding deterrence fixed. Optimal subsidies vary sharply across industries, consistent with the intuition that the national security case for industrial policy is strongest in a relatively narrow set of sectors. Priority sectors include guided missiles, propulsion units, and ammunition. Missiles and ammunition can be stockpiled but are expended in combat, so demand rises especially sharply during war. Optimal policy accommodates this demand by expanding industrial capacity during peace. Our exercise also suggests that industrial policy should depend on the conflict being deterred. A land war places greater emphasis on tank production, an air war on aircraft, and a naval war on shipbuilding. This implies that when wars are long, economic policymaking cannot be separated from the military assessment of how a war will be fought.

Related Literature This paper builds on early contributions to defense economics, including Hitch and McKean (1960); Schelling (1960, 1966). This early work put relatively little emphasis on the deterrence of long industrial wars. Part of the reason is that during the early Cold War the view was that wars were likely to be short and hence industrial capacity would not have much of a role. For example, in *The Economics of Defense in the Nuclear Age* (1960), Hitch and McKean argued that technological advances, such as nuclear weapons, had made wars requiring prolonged industrial mobilization relatively unlikely.[3] They therefore argued that policy should not focus on industrial capacity. During this time, economists therefore tended to focus on strategic aspects of deterrence relevant for short wars, such as where troops and nuclear weapons should be deployed.

Our contribution is to revisit questions of optimal economic policy in a setting where wars may last a long time. The Ukraine war shows that such wars remain relevant, even when a nuclear power is involved. We therefore formalize the links between industrial capacity, military power, and deterrence, and characterize the resulting rationale for industrial policy. Our framework contains both the Cold War view and the industrial wars view as special cases and thereby formalizes the intuition of Hitch and McKean that active support for industrial capacity is not needed to deter short wars. We also contribute by operationalizing the model through IDEM.[4]

Two related papers develop theoretical arguments for supporting the defense industrial base. The first, Kooi (2025), models the provision of national security as the provision of bargaining power. It uses this framework to study the strategic case for reducing exposure to war and trade shocks, since this exposure can give other states leverage in bargaining. The externality differs from ours: industrial policy strengthens a country's bargaining position by lowering its own cost of military or trade conflict.[5] That model can be operationalized with exogenous military shocks, whereas a central feature of IDEM is

[3]This emphasis on forces available at the outset was especially pronounced in analyses of nuclear war. Rowen (1960) argued that nuclear war would be decided before the year or more needed for mobilization, making peacetime forces much more important. NATO's 1957 strategic concept likewise prioritized existing forces, anticipating that nuclear destruction would severely restrict subsequent mobilization (NATO Military Committee, 1957). This emphasis also appeared in later Army doctrine: the 1976 edition of *FM 100-5: Operations* warned that initial combat could decide a future war, contrasting this with America's historical reliance on resources mobilized after hostilities began (Department of the Army, 1976).

[4]Early defense economists also sought to operationalize their ideas about deterrence. Schelling helped develop political-military wargames (Schelling, 1964), which continue to inform defense planning. For example, Shlapak and Johnson (2016) use wargaming to assess how reinforcing NATO's forces in the Baltics could improve military outcomes and strengthen deterrence.

[5]Models of bargaining and deterrence can generate similar incentives to alter the difference between the inside and outside options. Lower conflict costs can strengthen bargaining power or, in a deterrence model, make a country's commitment to fight if the status quo is challenged more credible. Our baseline takes this commitment as given, so this motive is absent. Section 3.5 extends our model to incorporate this additional strategic rationale and clarify how it differs from the one developed here.

that military production responds endogenously to industrial capacity. The second paper, Thompson (1979), provides a rationale for supporting the defense industrial base in a model in which wartime price controls lead to underinvestment during peacetime.

Our paper relates to the nascent geoeconomics literature (see Mohr and Trebesch (2025) for a recent review). This literature studies trade and financial relationships as a source of geopolitical power rather than military production. The combination of sticky capital investment taking place before bargaining is shared by several papers in this literature, including McLaren (1997); Adão and Kooi (2026); Becko and O'Connor (2025). Other work examines the links between military security and international economic relationships Alekseev and Lin (2025); Mayer et al. (2025). Further contributions to geoeconomics include Clayton et al. (2024, 2026) and Liu and Yang (2025).

We build on a literature in macroeconomics and public finance that studies investment, production, and government policy in response to war or military buildups. It typically treats these events as exogenous shocks and abstracts from the strategic decisions behind them. Empirical work in macroeconomics has used military spending to estimate fiscal multipliers (Ramey, 2011; Barro and Redlick, 2011; Nakamura and Steinsson, 2014). Recent work studies military buildups in their own right (see Ilzetzki (2025) for a review), examining their effects on military-goods prices Antonova et al. (2025) and their scale during major wars Marzian and Trebesch (2026).

We also build on a literature in political economy and international relations that uses game-theoretic models to study war or military buildups as endogenous outcomes of countries' assessments of the costs and benefits of conflict. It typically abstracts from private-sector investment decisions and from features of the economy such as input–output networks. Building on Schelling (1960, 1966), it recognizes that because war is costly, national security policy is often about preventing conflict through deterrence. Work in this tradition includes Mearsheimer (1983); Baliga et al. (2020); Baliga and Sjöström (2020); Luo and Wolitzky (2025); Herrera et al. (2022); Di Lonardo and Tyson (2022, 2026).

We also contribute to the literature on industrial policy (reviewed in Juhász et al. (2024)) by providing a theoretical foundation for industrial policy in the defense industrial base. This literature has studied the empirical effects of defense-related policies, including military procurement, public R&D, and direct investment in manufacturing capacity Lichtenberg (1988); Gross and Sampat (2023); Ilzetzki (2024); Moretti et al. (2025); Garin and Rothbaum (2025), but formal arguments for prioritizing the defense industrial base in peacetime remain less developed.

2 A Model of Deterrence through Industrial Capacity

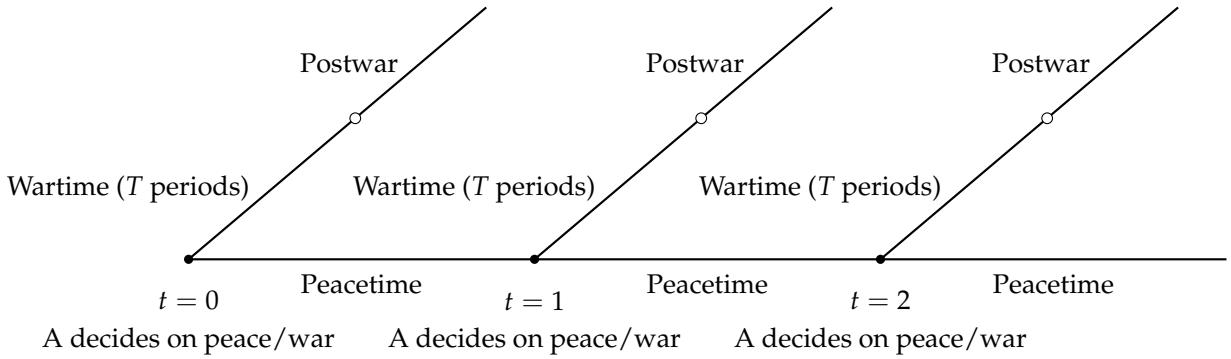
We begin with a framework for studying industrial capacity and deterrence. To establish our main insights, we use the simplest setting that both accommodates the extensions in Section 3.5 and sets the stage for the quantitative model used in Sections 4 and 5.

Overview There are two countries, Sovereign and Adversary, indexed by $i \in \{S, A\}$, each inhabited by a representative agent. Time is discrete and infinite, with periods $t = 0, 1, \ldots$. Both agents value consumption and a public good, interpreted as territory. This good is non-rival within a country, but each unit of territory can only be enjoyed by the citizens of one country, and hence it is rival between countries.

The economy begins in peace, during which the allocation of territory remains at the status quo. At the beginning of each peacetime period, Adversary decides whether to maintain peace or start a war. A war lasts one or two periods, $T \in \{1, 2\}$, referred to as short and long wars, respectively. Its duration is exogenous and known to both countries. At the end of the war, territory is reallocated according to the countries' relative military strength over the conflict. This strength reflects initial military stocks and additional equipment accumulated during the war. Long wars allow newly produced equipment to affect battlefield outcomes; short wars do not. Duration is therefore defined relative to production timelines. War can occur only once: afterward, economic activity continues, but no further war is possible.

Figure 1 illustrates this timing. We focus on equilibria in which Sovereign sustains peace, so only the horizontal peacetime path is realized. The wartime and postwar branches describe the off-equilibrium continuations that determine Adversary's incentive to attack.

Figure 1: Timing of peace, war, and the postwar period.



2.1 Economic Block

We begin by describing the economic block of the model, which is common to both countries. Households supply labor and own firms, while the government procures military goods and can subsidize capital in the defense industrial base. We denote the geopolitical state by $z_t \in \{P, C\}$, where $z_t = P$ if no war has begun and $z_t = C$ once a war has begun, including the postwar period. In what follows, all equilibrium variables are, in general, functions of the history $z^t = (z_0, z_1, \ldots, z_t)$, but whenever no confusion is possible, we leave this dependence implicit.

Preferences The representative agent in each country has preferences given by

$$U^i = \sum_{t=0}^{\infty} \beta^t \left(G_t^i + c_t^i \right), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and $\{c_t^i\}_{t=0}^{\infty}$ is the sequence of consumption. Here G_t^i is the public good introduced above, which is not chosen by households but is instead determined by geopolitics.

Capital Accumulation There are two capital stocks in this economy. There is a stock of military capital, M_t^i, which represents the strength of the armed forces and depreciates at rate δ_M. There is also capital in the defense industrial base, K_t^i, which serves as an input into the production of military capital and depreciates at rate δ_K. Let m_t^i denote government purchases of new military equipment and i_t^i investment in industrial capital during period t. The two capital stocks therefore evolve according to

$$M_{t+1}^i = (1 - \delta_M) M_t^i + m_t^i \tag{2}$$

$$K_{t+1}^i = (1 - \delta_K) K_t^i + i_t^i. \tag{3}$$

Both stocks are predetermined at the beginning of period t. Purchases and investment during period t augment the stocks available in period $t + 1$.

Production There are two production technologies in this economy. Output of the numeraire good is denoted by $y_{n,t}^i$ and is produced through a linear technology that uses labor $l_{n,t}^i$ as input. The defense sector produces military goods, denoted by $y_{m,t}^i$, which are used to accumulate military capital. Military goods are produced using a Cobb–Douglas technology with $\alpha \in (0, 1)$ that takes K_t^i and labor, $l_{m,t}^i$, as inputs. We normalize produc-

tivity in military goods production to one. The two technologies are given by

$$y^i_{n,t} = l^i_{n,t} \tag{4}$$

$$y^i_{m,t} = \left(K^i_t\right)^{\alpha} \left(l^i_{m,t}\right)^{1-\alpha}. \tag{5}$$

The producers of the numeraire good solve a static problem in which they maximize profits by hiring labor:

$$\max_{y^i_{n,t}, l^i_{n,t}} \left\{ p^i_{n,t} y^i_{n,t} - w^i_t l^i_{n,t} \,\middle|\, (4),\ l^i_{n,t} \geq 0 \right\}. \tag{6}$$

The defense industrial base maximizes profits by choosing investment and labor to produce military goods. It takes as given the price of military goods, $p^i_{m,t}$, and the subsidy on its capital stock, s^i_t, which the government uses to affect investment decisions. The subsidy is a payment of s^i_t per unit of installed capital in period t. Given its initial capital stock K^i_0, its optimization problem is therefore

$$\max_{\{y^i_{m,t}, l^i_{m,t}, i^i_t, K^i_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \frac{1}{R^i_t} \left(p^i_{m,t} y^i_{m,t} + s^i_t K^i_t - w^i_t l^i_{m,t} - p^i_{n,t} i^i_t \right) \,\middle|\, (3),\ (5),\ l^i_{m,t} \geq 0,\ i^i_t \geq 0 \right\}. \tag{7}$$

The government raises revenue to fund the subsidies it pays to the defense industrial base and to procure military goods. It uses a lump-sum tax to balance its budget period by period and starts with no initial debt. Its budget constraint is therefore given by

$$T^i_t = p^i_{m,t} m^i_t + s^i_t K^i_t. \tag{8}$$

Market Clearing The numeraire good can be used for consumption or investment in the defense industrial base. The military good is used only to accumulate military capital. The economy is endowed with $\bar{l}^i$ units of labor, which are allocated between the numeraire sector and the defense sector. The three market-clearing conditions are therefore given by

$$c^i_t + i^i_t = y^i_{n,t} \tag{9}$$

$$m^i_t = y^i_{m,t} \tag{10}$$

$$l^i_{n,t} + l^i_{m,t} = \bar{l}^i. \tag{11}$$

Competitive Equilibrium Households choose consumption to maximize utility subject to their budget constraint, taking as given the price of the numeraire good $p^i_{n,t}$, the wage w^i_t, firm profits Π^i_t, and the lump-sum tax T^i_t levied by the government. They have access to a risk-free bond in zero net supply, with a rate of return r^i_t between periods t and $t+1$. Let $R^i_t = \prod_{u=0}^{t-1}(1+r^i_u)$ denote the cumulative gross return from date 0 to date t, with $R^i_0 = 1$. The household's problem is

$$\max_{\{c^i_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t c^i_t \;\middle|\; \sum_{t=0}^{\infty} \frac{1}{R^i_t} \left(p^i_{n,t} c^i_t - w^i_t \bar{l}^i - \Pi^i_t + T^i_t \right) = 0 \right\}. \tag{12}$$

Definition 1 (Competitive Equilibrium)**.** *Taking as given the sequence of geopolitical states $\{z_t\}_{t=0}^{\infty}$, the sequence of government purchases and subsidies $\{m^i_t, s^i_t\}_{t=0}^{\infty}$, and the initial capital stocks K^i_0 and M^i_0, a competitive equilibrium in country i is a sequence of prices $\{r^i_t, p^i_{n,t}, p^i_{m,t}, w^i_t\}_{t=0}^{\infty}$, quantities $\{c^i_t, y^i_{n,t}, y^i_{m,t}, i^i_t, K^i_t, M^i_t, l^i_{n,t}, l^i_{m,t}\}_{t=0}^{\infty}$, and taxes and profits $\{T^i_t, \Pi^i_t\}_{t=0}^{\infty}$ such that households and firms solve their optimization problems, markets clear, and the government budget constraint holds in every period.*

2.2 Deterrence Block

We now describe the deterrence block. We use t to index calendar time and write peacetime quantities without a regime superscript. When considering a war that begins at date t, we index its continuation by $\tau = 0, 1, \ldots$, where $\tau = 0$ is the first period of war. For a generic variable X, we write $X^{i,C}_\tau$ for its value τ periods after the outbreak, including the postwar period. Since capital stocks are predetermined when Adversary decides whether to attack, the initial stocks in this continuation satisfy $K^{i,C}_0 = K^i_t$ and $M^{i,C}_0 = M^i_t$.

The Allocation of the Public Good Countries are endowed with an initial allocation of the public good, $\bar{G}^i$. During peacetime, the allocation of the public good is fixed at its initial level. This implies that

$$G^i_t = \bar{G}^i. \tag{13}$$

The allocation of the public good can only be changed through war. At the end of the war, the public good is reallocated based on the parties' relative military capacity throughout the war. The duration of the war, $T \in \{1, 2\}$, is exogenous and known to both parties. The technology is linear, and the parameter $\chi > 0$ captures the effectiveness of military power in obtaining territory. The new allocation applies from the final period of war onward and

remains fixed thereafter:

$$G_{\tau}^{i,C} = \begin{cases} \bar{G}^i, & \tau < T-1, \\ \bar{G}^i + \chi \sum_{u=0}^{T-1} \left(M_u^{i,C} - M_u^{-i,C} \right), & \tau \geq T-1, \end{cases} \tag{14}$$

where $-i$ denotes the other country.

Adversary's Policy Rules Adversary's government is assumed to follow simple policy rules that keep its military and industrial capital stocks at their initial levels, denoted by $\bar{M}^A$ and $\bar{K}^A$, respectively. It replaces depreciated military equipment and industrial capital in every period, during peace, war, and the postwar period.

The reason for restricting attention to such simple rules is that the focus is on the strategic interaction between Sovereign's military procurement and industrial policies, on the one hand, and Adversary's aggression decision, on the other. Allowing for richer behavioral responses by Adversary would introduce additional strategic interactions. Section 3.5 explains how to incorporate these and discusses their implications for policy.

Adversary's Challenge Decision At the beginning of any peacetime period, Adversary can challenge the status-quo allocation of territory by starting a war. Let $W_t^{A,P}(\mathbf{x}_t)$ denote its discounted welfare from maintaining peace at date t, and let $W_t^{A,C}(\mathbf{x}_t)$ denote its discounted welfare from starting a war at that date. Both values include consumption and public-good flows from date t onward. Here $\mathbf{x}_t = \{G_t^i, K_t^i, M_t^i\}_{i \in \{S,A\}}$ denotes the vector of state variables at the time of the decision. Choosing peace at date t leaves open the possibility of starting a war at a later date; the superscript P therefore refers to the current decision, rather than a commitment to permanent peace.

Let η_t^A denote Adversary's additional political payoff from going to war. Adversary starts a war whenever

$$W_t^{A,C}(\mathbf{x}_t) + \eta_t^A > W_t^{A,P}(\mathbf{x}_t). \tag{15}$$

A higher η_t^A therefore makes Adversary more willing to attack. When indifferent, it maintains the peace.

2.3 Deterrence Equilibrium

We consider equilibria in which Adversary maintains the peace at every point in time. We define deterrence as

$$\mathcal{D}_t(\mathbf{x}_t) \equiv W_t^{A,P}(\mathbf{x}_t) - W_t^{A,C}(\mathbf{x}_t). \tag{16}$$

This measures Adversary's welfare advantage from maintaining peace, before including the additional political payoff from attacking. Adversary is deterred when this advantage is at least as large as η_t^A. We therefore define a deterrence equilibrium as follows.

Definition 2 (Deterrence equilibrium)**.** *A deterrence equilibrium is a deterministic sequence of quantities, prices, subsidies, taxes, and profits that (i) is consistent with the competitive equilibrium and (ii) sustains the peace by satisfying the deterrence constraint*

$$\mathcal{D}_t(\mathbf{x}_t) \geq \eta_t^A, \quad \forall t. \tag{17}$$

There may be many allocations that satisfy these equilibrium conditions.[6] Specifically, there may be several feasible combinations of military and defense-industrial capital that satisfy the deterrence constraint (17) and can be implemented in a competitive equilibrium. Section 3 studies which policies Sovereign needs in order to attain the deterrence equilibrium that maximizes domestic welfare. First, however, it is useful to characterize the deterrence constraint more explicitly.

2.4 Simplifying the Deterrence Constraint

Our definition of equilibrium is expressed in terms of $\mathcal{D}_t(\mathbf{x}_t)$. As will become clear in the next section, the characterization of optimal subsidies applies to general deterrence functions. However, our comparative statics use the microfoundations of $\mathcal{D}_t(\mathbf{x}_t)$. In this section, we show how our assumptions simplify this function.

Adversary's Welfare during Peace We begin by deriving an expression for Adversary's welfare during peace. Because Adversary maintains its military and industrial capital stocks at $\bar{M}^A$ and $\bar{K}^A$, its consumption is constant. It equals the total labor endowment minus the labor used to produce replacement military equipment and industrial capital:

$$c_t^A = \bar{c}^A \equiv \bar{l}^A - \delta_K \bar{K}^A - \left(\frac{\delta_M \bar{M}^A}{(\bar{K}^A)^\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{18}$$

[6] We assume that a feasible deterrence equilibrium exists even in the absence of industrial policy.

In a deterrence equilibrium, Adversary expects peace to continue at every future date. Its peacetime welfare is therefore

$$W_t^{A,P} = \frac{\bar{G}^A + \bar{c}^A}{1-\beta}. \tag{19}$$

Sovereign's Wartime Problem Adversary's welfare if it chooses war depends in part on how much military equipment Sovereign produces during the conflict. We therefore begin by studying Sovereign's optimal policy during war.

Consider a war beginning at date t. Sovereign enters the conflict with capital stocks K_t^S and M_t^S and the status-quo territorial allocation $\bar{G}^S$. Substituting the conflict technology (14) into the household objective function yields the following policy problem:

$$\max_{\substack{\{c_\tau^{S,C}, y_{n,\tau}^{S,C}, y_{m,\tau}^{S,C}, l_{n,\tau}^{S,C}, l_{m,\tau}^{S,C}, \\ i_\tau^{S,C}, m_\tau^{S,C}, K_{\tau+1}^{S,C}, M_{\tau+1}^{S,C}\}_{\tau=0}^{\infty}}} \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left(c_\tau^{S,C} + \bar{G}^S \right) + \frac{\chi \beta^{T-1}}{1-\beta} \sum_{\tau=0}^{T-1} \left(M_\tau^{S,C} - \bar{M}^A \right) \,\middle|\, (2)\text{–}(11) \right\}. \tag{20}$$

The constraints are evaluated along the conflict continuation, with initial stocks inherited from date t and all allocations subject to non-negativity constraints. Since $\bar{G}^S$ and $\bar{M}^A$ enter the objective additively and separably, Sovereign's optimal military stocks depend only on its inherited capital stocks, holding parameters fixed. We denote this response by $M_\tau^{S,C}(K_t^S, M_t^S)$.

Adversary's Welfare during War Having characterized Sovereign's military stocks as a function of its inherited capital stocks, we can now characterize Adversary's welfare during war. Adversary maintains the same consumption during peace and conflict, so $c_\tau^{A,C} = \bar{c}^A$. Substituting Sovereign's military stocks into the conflict technology (14) gives Adversary's welfare if it attacks at date t:

$$W_t^{A,C} = \frac{\bar{G}^A + \bar{c}^A}{1-\beta} + \frac{\chi \beta^{T-1}}{1-\beta} \sum_{\tau=0}^{T-1} \left(\bar{M}^A - M_\tau^{S,C}(K_t^S, M_t^S) \right). \tag{21}$$

The Simplified Deterrence Constraint We now combine (19) and (21) to simplify the deterrence constraint. The common consumption and status-quo territorial terms cancel. Since the capital stocks available when war begins are predetermined, the wartime response is evaluated at Sovereign's peacetime stocks K_t^S and M_t^S. Keeping Adversary's fixed military stock $\bar{M}^A$ implicit as a parameter, we can therefore write deterrence as

$$\mathcal{D}_t(K_t^S, M_t^S) = \frac{\chi \beta^{T-1}}{1-\beta} \sum_{\tau=0}^{T-1} \left(M_\tau^{S,C}(K_t^S, M_t^S) - \bar{M}^A \right), \tag{22}$$

where $M_0^{S,C} = M_t^S$ for both short and long wars. For long wars,

$$M_1^{S,C} = (1-\delta_M)M_t^S + m_0^{S,C}(K_t^S), \quad m_0^{S,C}(K) = \min\left\{ \left[\tfrac{\chi\beta(1-\alpha)}{1-\beta}\right]^{\frac{1-\alpha}{\alpha}} K,\ K^\alpha(\bar{l}^S)^{1-\alpha} \right\}. \tag{23}$$

The first term in the minimum describes production when some labor remains in the numeraire sector; the second is the maximum output attainable with full labor mobilization. Military output as a function of capital is therefore continuous but kinked at a single point. For expositional ease, the main text focuses on allocations at which the deterrence function is differentiable, without further comment; the proofs also treat the kink.[7]

Deterrence is increasing in the amount of military power deployed throughout the war.[8] The deterrence constraint used going forward is therefore

$$\mathcal{D}_t(K_t^S, M_t^S) \geq \eta_t^A. \tag{24}$$

2.5 Discussion

Before turning to optimal economic policy, we briefly comment on some of the modelling assumptions.

The Asymmetries of Deterrence In our model, there is a clear asymmetry in the security problem for the two countries. Adversary is the challenger to the status quo, whereas Sovereign is its defender. We view this as a useful simplification that allows us to develop the strategic argument for industrial policy. But such asymmetries are also quite natural from a historical perspective. When the balance of geopolitical power changes—for example, because the economies of rising powers grow faster than those of existing powers (Kennedy, 1987)—existing powers find themselves trying to preserve a status quo that favors them for historical reasons. The asymmetry thus arises endogenously from changes in the balance of power. We view this as a useful description of the security problem facing Western countries seeking to deter challenges to the geopolitical status quo by China and Russia.

[7] We could instead restrict attention to cases in which the wartime labor constraint is slack. The strictly binding case is extreme: all labor is devoted to military production, yet Sovereign would like to expand military output further. We retain this case to clarify the theoretical point in Section 3.2 that capital is not merely a commitment device: it continues to strengthen deterrence even under full labor mobilization, the wartime allocation Sovereign would promise under full commitment.

[8] The model can be extended to allow Adversary's stocks to follow exogenous paths that differ between peace and conflict. These paths would change the level of $\mathcal{D}_t$ but leave its marginal effects with respect to K_t^S and M_t^S unchanged. Such shifts in the deterrence constraint can be represented through changes in η_t^A.

Policy Instruments We study a specific form of industrial policy in the form of subsidies to the capital stock. In these types of models, there are in general many policies the government could use to implement its desired allocation of capital, including investment subsidies or direct government control of the defense industrial base. The subsidies are therefore best interpreted as wedges that clarify when the government should deviate from a competitive equilibrium without industrial policy rather than as a prescription for the use of a particular policy instrument.

Deterrence and Beliefs We have emphasized that the duration of war matters for deterrence. In the baseline model, countries have rational expectations in the sense that they agree on T, and their beliefs coincide with the true duration of war. But because war never occurs in a deterrence equilibrium, what matters for deterrence is how Adversary expects a war to unfold if it attacks. Sovereign must therefore take into account Adversary's beliefs about the duration of war, which may not always coincide with its true duration. For example, Mearsheimer (1983) argues that deterrence is more likely to fail when an adversary believes that a war will be short—that its troops will be home by Christmas. Misperceptions can also go in the opposite direction: leaders may expect wars to be long even when they turn out to be short.[9] The argument for industrial policy does not require Adversary's beliefs about war duration to be correct. What matters is that Adversary assigns some positive probability to a war lasting long enough for Sovereign's wartime production to affect the outcome. Of course, when Adversary believes that wars are likely to be short, it places less weight on Sovereign's ability to produce additional equipment, weakening the deterrent value of industrial capital.

3 Optimal Economic Policy to Sustain the Peace

In this section, we study how the government should use procurement and industrial policy to maximize domestic welfare while sustaining peace through deterrence. After stating the peacetime planning problem, we explain when and why capital in the defense industrial base contributes to deterrence. We then characterize the optimal policy and explain why, in the absence of policy, the market and the planner solve different cost-minimization problems. Finally, we show why optimal industrial policy can encourage investment in technologies that allow military production to scale during war. Throughout we focus on the case where investment and procurement are strictly positive.

[9] For example, German economic planners were preparing during the fall of 1939 for a long, drawn-out war against France, yet the campaign ended after only six weeks. See chapter 10 in Tooze (2007).

3.1 Sovereign's Peacetime Planning Problem

The central question is how Sovereign should choose investment in military capital and in the defense industrial base in order to attain the deterrence equilibrium that maximizes domestic welfare. Given initial capital stocks K_0^S and M_0^S, we therefore focus on the following planning problem:

$$\max_{\{c_t^S, y_{n,t}^S, y_{m,t}^S, l_{n,t}^S, l_{m,t}^S, i_t^S, m_t^S, K_{t+1}^S, M_{t+1}^S\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left(\bar{G}^S + c_t^S\right)$$

s.t. (i) Technology and market-clearing constraints (2)–(11), for all t,

(ii) Deterrence constraint (24), for all $t \geq 1$,

(iii) Non-negativity constraints, for all t.

The planner chooses only peacetime allocations. In the deterrence equilibria we study, wars are off the equilibrium path, and the planner is not allowed to commit to off-path allocations. If war begins, Sovereign solves the wartime problem in Section 2.4 given its inherited capital stocks. This response is already incorporated into the deterrence constraint.

Note that this is a primal planning problem. The planner chooses the entire sequence of peacetime allocations directly. Household and firm optimization constraints are not imposed explicitly. Instead, we later verify that the allocations solving this problem can be decentralized as a competitive equilibrium using procurement and capital subsidies.

3.2 The Optimal Deterrence Decision

In this section, we study the optimal investment decisions and explain when and why capital in the defense industrial base contributes to deterrence.

Optimal Investment At an interior peacetime allocation, the optimality conditions for investment in military capital and defense capital are, respectively,

$$\lambda_{m,t}^S = \beta \left(\gamma_{t+1}^S \frac{\partial \mathcal{D}_{t+1}}{\partial M_{t+1}^S} + (1 - \delta_M) \lambda_{m,t+1}^S \right), \tag{25}$$

$$1 = \beta \left(\gamma_{t+1}^S \frac{\partial \mathcal{D}_{t+1}}{\partial K_{t+1}^S} + (1 - \delta_K) + \lambda_{m,t+1}^S \alpha \left(K_{t+1}^S\right)^{\alpha-1} \left(l_{m,t+1}^S\right)^{1-\alpha} \right), \tag{26}$$

where γ_t^S is the multiplier on (24) and $\lambda_{m,t}^S$ is the multiplier on (2), both at date t.

Equation (25) characterizes optimal investment in the stock of military capital. Its value is determined by two forces: its deterrent value next period, given by the shadow value of deterrence γ_{t+1}^S multiplied by its marginal contribution to deterrence, and the value of the equipment that remains after depreciation. Equation (26) characterizes optimal investment in the defense industrial base. The planner values defense capital both because it contributes directly to deterrence and because it raises military-goods output, as captured by the final term. As we show below, the productive role of defense capital during peace is reflected in market prices, whereas its role as a deterrent is not.

The Deterrent Value of Military and Industrial Capacity Before turning to the analysis of optimal policy, it is useful to clarify the determinants of the deterrent value of capital.

Proposition 1 (Capital and Deterrence)**.** *Suppose wars are short ($T = 1$). Then the marginal deterrent values of military and industrial capacity at a possible attack date t are*

$$\frac{\partial \mathcal{D}_t}{\partial M_t^S} = \frac{\chi}{1-\beta} > 0, \qquad \frac{\partial \mathcal{D}_t}{\partial K_t^S} = 0.$$

Suppose instead that wars are long ($T = 2$) and some labor remains in numeraire production in the first period of war. Then the deterrent values of military and industrial capacity are

$$\frac{\partial \mathcal{D}_t}{\partial M_t^S} = \frac{\chi\beta}{1-\beta}\Big(1 + (1-\delta_M)\Big) > 0, \qquad \frac{\partial \mathcal{D}_t}{\partial K_t^S} = \left(\frac{\chi\beta}{1-\beta}\right)^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} > 0. \tag{27}$$

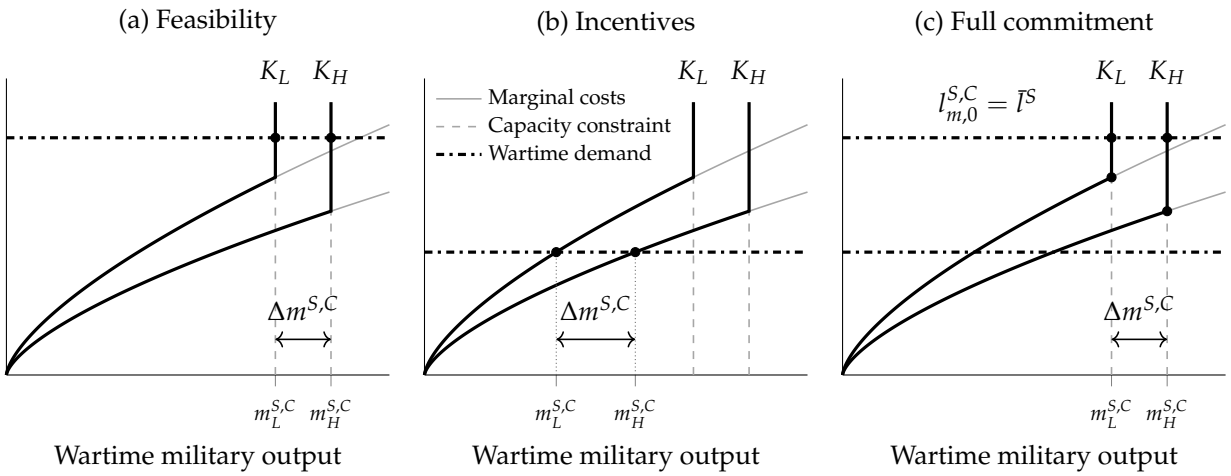
Suppose instead that wars are long ($T = 2$) and the labor constraint binds strictly in the first period of war, so all labor is allocated to military production. Then the deterrent values of military and industrial capacity are

$$\frac{\partial \mathcal{D}_t}{\partial M_t^S} = \frac{\chi\beta}{1-\beta}\Big(1 + (1-\delta_M)\Big) > 0, \qquad \frac{\partial \mathcal{D}_t}{\partial K_t^S} = \frac{\chi\beta}{1-\beta}\alpha \left(K_t^S\right)^{\alpha-1} \left(\bar{l}^S\right)^{1-\alpha} > 0. \tag{28}$$

Proof. Found in Appendix A.2. □

The proposition makes three points. First, military capacity deters in both short and long wars. Second, holding the initial military stock fixed, industrial capacity acts as a deterrent only when wars are long. The reason is that, in this model, wartime production can affect battlefield outcomes only if there is time to build before the war ends. Industrial capacity can still contribute to building the peacetime military stock before a short war. Third, the deterrent value of the defense industrial base depends on whether wartime production is limited by incentives or feasibility. We discuss these cases next.

Figure 2: Industrial capital and wartime military production



Note: All panels use the same supply curves; $m_L^{S,C}$ and $m_H^{S,C}$ denote wartime output at $K_L < K_H$. Under full commitment, Sovereign promises full labor mobilization at either demand level. When wartime demand is sufficiently low, the marginal cost of wartime production exceeds its marginal benefit at the full commitment allocation, as illustrated in panel (c).

Incentive Compatibility versus Feasibility Why does capital contribute to deterrence? In this model, it does so during long wars because, holding initial military stocks fixed, an increase in K raises wartime military production $m^{S,C}(K)$ and thereby reduces Adversary's gain from attacking. There are two reasons why capital can raise wartime production, illustrated in Figure 2.

First, capital can relax a *feasibility constraint*, as shown in panel (a). Sovereign would like to expand military production further, but all available labor is already allocated to it. Production is therefore limited by the capacity of the defense industrial base. Raising capital from K_L to K_H allows the same labor endowment to produce more military equipment, shifting the capacity constraint outward and strengthening deterrence.

Second, capital can relax a *wartime incentive compatibility constraint* arising from limited commitment. During peace, Sovereign would like to commit to allocating all available labor to military production if war begins. The prospect of a larger wartime buildup strengthens deterrence, while the costs of carrying it out are never incurred in a deterrence equilibrium. Once war begins, however, Sovereign must weigh the military benefits of production against the loss of consumption. It may therefore prefer to retain some labor in numeraire production rather than carry out the promised buildup. This difference between what Sovereign would like to promise during peace and what it would choose during war is the time-inconsistency problem.

Panel (b) illustrates this case: wartime demand intersects the upward-sloping part

of the supply curve, so some labor remains in numeraire production. Here, an increase in capital raises wartime military output through two effects. First, it raises output for a given allocation of labor, just as in the feasibility-constrained case. Second, it raises the marginal product of military labor, inducing Sovereign to reallocate additional labor toward military production. This latter effect brings the labor allocation closer to the full-commitment solution in which all available labor is mobilized. It is this crowding-in of military labor that we refer to as relaxing the wartime incentive compatibility constraint.

Finally, panel (c) shows that capital continues to deter even when full commitment removes the wartime incentive compatibility constraint and the associated time-inconsistency problem. Sovereign promises full labor mobilization at either demand level, raising wartime production relative to panel (b) but leaving it unchanged relative to panel (a). With labor already fully mobilized, additional capital cannot crowd in more military labor. It nevertheless raises the output that the same labor endowment can produce. Capital therefore continues to strengthen deterrence by relaxing the feasibility constraint.

3.3 Optimal Industrial Policy

In this section, we first provide a sufficient statistic for optimal industrial policies. We then explain why industrial policy can be needed to efficiently deter long but not short wars. Lastly, we clarify the market failure by explaining why procurement policy alone does not minimize the cost of deterrence during long wars.

Optimal Subsidies We begin by studying the firm's decision to invest in capital in the defense industrial base. At an interior allocation, its first-order condition for optimal investment can be expressed as

$$1 = \beta\Big((1-\delta_K) + p^S_{m,t+1}\alpha(K^S_{t+1})^{\alpha-1}(l^S_{m,t+1})^{1-\alpha} + s^S_{t+1}\Big). \tag{29}$$

It shows that, in the absence of industrial policy, the firm invests because it increases its output, which it gets to sell at market prices. It does not consider the effect of its investment on the deterrence constraint. Each competitive firm takes prices and government procurement as given. Slackening the deterrence constraint reduces the resources needed to sustain deterrence and benefits all households, but the firm receives no payment for this contribution. At the allocation being implemented, $p^S_{m,t} = \lambda^S_{m,t}$, so comparison with (26) gives the following result.

Proposition 2. *An optimal deterrence equilibrium can be implemented by choosing procurement optimally and setting subsidies equal to*

$$s_t^S = \gamma_t^S \frac{\partial \mathcal{D}_t}{\partial K_t^S}. \tag{30}$$

Proof. Delegated to Appendix A.3. □

The expression shows that industrial policy is used because capital has a strategic value as a deterrent. The subsidy equals its marginal contribution to deterrence multiplied by the shadow value of deterrence. The other reason for a firm to value capital—because it uses it to produce output—is valued appropriately because it receives the market price for that output and hence does not appear in the expression.

But why does the market not price capital's contribution to deterrence? Additional industrial capital allows the government to sustain the same level of deterrence with less military procurement. The resulting resource savings benefit households, but individual firms receive no payment for creating them: they take procurement prices and lump-sum taxes as given.[10] This argument does not depend on whether lump-sum taxes are levied on households or firms. Even if these taxes were levied on firms, each firm would be too small to internalize its contribution to aggregate procurement savings through its own tax bill.[11]

We now turn to a key comparative static. The next proposition shows how optimal industrial policy depends on the length of the war.

Proposition 3. *Suppose wars are short ($T = 1$). Then the planner does not use industrial policy, $s_t^S = 0$. Suppose instead wars are long ($T = 2$) and the deterrence constraint binds, $\gamma_t^S > 0$. Then optimal subsidies are positive, $s_t^S > 0$, regardless of whether it is the incentive compatibility or feasibility constraint that binds.*

Proof: Immediate from the above propositions.

The proposition shows that whether there is a role for industrial policy is endogenous to the nature of the war. When wars are short, the defense industrial base does not add to deterrence, holding the initial military stock fixed, and so industrial policy is not needed.

[10] Capital subsidies also require revenue. But because households own the firms in the defense industrial base and taxation is lump-sum, the subsidy payments themselves are domestic transfers.

[11] When war can occur, as in the extension discussed in Section 3.5, the benefit of additional capital need not appear as lower procurement expenditure. At a given level of procurement, additional capital improves Sovereign's wartime production capacity, making an attack less attractive to Adversary and thereby reducing the probability of war. Individual firms do not internalize this contribution to lower war risk either. At an interior optimum, the government is indifferent, to first order, between these two uses of additional capital: otherwise, it could adjust procurement and improve welfare.

When wars are long and the deterrence constraint has a positive shadow value, it does, and hence industrial policy is needed.[12]

On the Difference between the Market and the Planner Lastly, we explain why the competitive equilibrium can be inefficient in the absence of industrial policy. We show that the planner minimizes the cost of deterrence while the market minimizes the cost of military equipment. These two problems happen to coincide when wars are short but not when wars are long.

We illustrate this point by studying the steady state of the model when $\beta \to 1$.[13] This case is convenient because the steady-state problems of both the planner and the market are effectively static. The solution of the optimal static problem then coincides with the limiting steady-state allocation of the dynamic model.[14] We then obtain the following result.

Proposition 4. *Consider a steady state of the Ramsey problem when $\beta \to 1$ with a positive shadow value of deterrence. The input choices in the defense industrial base of the competitive equilibrium in the absence of industrial policy minimize the cost of producing the steady-state level of military equipment. The planner instead minimizes the cost of providing deterrence. These problems coincide when wars are short but not when they are long. This implies that, conditional on government procurement, there is too little capital and hence too much labor used in military production when wars are long.*

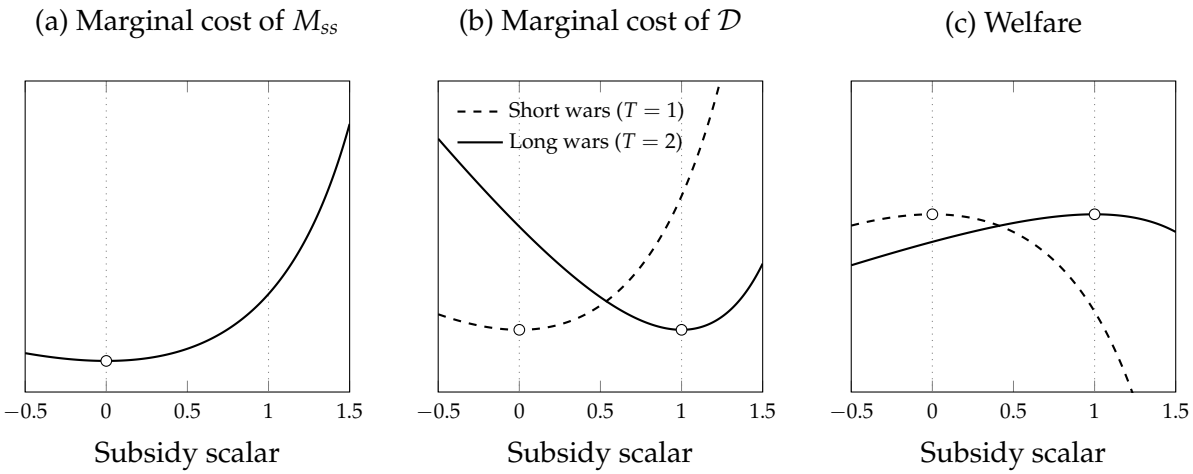
The intuition is best explained graphically. Figure 3(a) plots the steady-state marginal cost of maintaining a larger military stock. This cost is minimized when the government does not use industrial policy to distort the private sector's choice of inputs. Panel (b) instead plots the steady-state marginal cost of providing additional deterrence. For short wars, this cost is also minimized without industrial policy. For long wars, however, it is minimized with positive subsidies, because industrial capacity itself contributes to deterrence. Panel (c) shows the corresponding welfare implications: welfare is maximized at the subsidy level that minimizes the cost of providing deterrence.

[12]The deterrence constraint may not always bind. The reason it may not is that an economy could start with very high M_0^S and K_0^S. In an optimal steady state with positive military capital, the deterrence constraint always binds because otherwise a country could lower the steady-state military capital stock and enjoy higher consumption.

[13]To ensure that the value of military equipment during conflict does not explode as $\beta \to 1$, the marginal benefit of military equipment is rescaled accordingly, i.e., $\chi = (1-\beta)\hat{\chi}$ for some $\hat{\chi} > 0$.

[14]The logic is analogous to the neoclassical growth model, where the Ramsey capital stock coincides with the consumption-maximizing capital stock when $\beta \to 1$.

Figure 3: Deterrence equilibria and optimal subsidies



Note: The subsidy scalar multiplies the planner's optimal subsidy for long wars: 1 is the optimum and 0 implies no subsidies. Panel (a) plots the steady-state marginal cost of M_{ss}. Panels (b) and (c) plot the steady-state marginal cost of $\mathcal{D}$ and welfare for short and long wars. Cost curves are normalized at their own minima and welfare curves at their own maxima.

3.4 Encouraging the Capacity to Scale

As policymakers revisit the role of industrial policy in the defense industrial base, they have emphasized investments that allow production to scale during war. We now study the sense in which policy should encourage this capacity to scale. In the baseline model, only a single technology is used in the defense industrial base, so there is no sense in which policy can explicitly encourage "scalability". We therefore introduce a second technology to capture the idea that some production technologies allow output to expand more readily than others. Our notion of scalability builds on Murphy and Topel (2013), who think of a technology as scalable if it has a high output elasticity with respect to a flexible input. We augment this idea by also modelling the elasticity of supply of that input. For a technology to be scalable in practice, output must respond strongly to flexible inputs, and those inputs must be available to the defense sector without sharply increasing their cost.

The baseline model is therefore extended by allowing military goods to be produced using two technologies $g \in \{1, 2\}$. Each technology has a Cobb–Douglas production function with constant returns to scale, $m_g = K_g^{\alpha_g} l_g^{1-\alpha_g}$, and total output of the military good is $m = \sum_g m_g$. One can think of the two technologies as representing different ways to produce the same equipment, or as different types of equipment that serve a similar function on the battlefield. Labor is supplied separately to each technology according to $w_g(l_g) = l_g^{1/\varepsilon_{\ell,g}}$, where $\varepsilon_{\ell,g}$ is the sectoral labor supply elasticity. The upward-sloping la-

bor supply curves make the marginal cost of production rise as each technology expands, allowing both technologies to be used despite constant returns in production.[15]

We begin by establishing that the key results from the previous sections generalize.

Proposition 5. *When there are two technologies, optimal investment subsidies are given by*

$$s^S_{g,t} = \gamma^S_t \frac{\partial \mathcal{D}_t}{\partial K^S_{g,t}}. \tag{31}$$

If the deterrence constraint has a positive shadow value, the planner chooses strictly positive investment subsidies for each active technology in long wars but does not use them in short wars.

Proof. Found in Appendix A.4. □

Next, we ask how investment subsidies depend on the capacity to scale production. In this extension, this capacity depends on both the output elasticity of labor and the labor supply elasticity. Holding capital fixed, the short-run supply elasticity of technology g is

$$\varepsilon_g \equiv \left.\frac{\partial \log m_g}{\partial \log p_m}\right|_{K_g} = \frac{1-\alpha_g}{\alpha_g + 1/\varepsilon_{\ell,g}}. \tag{32}$$

A higher output elasticity of labor, $1-\alpha_g$, or a higher labor supply elasticity, $\varepsilon_{\ell,g}$, therefore makes production more scalable in the short run. The supply elasticity is constant, so the same expression applies during peace and war.

Proposition 6. *Consider optimal subsidies in the steady state when wars are long, both technologies are active, and the deterrence constraint has a positive shadow value. Suppose that military production rises during conflict relative to the peacetime steady state and that technology 1 is more scalable, $\varepsilon_1 > \varepsilon_2$. In this case the planner subsidizes technology 1 relative to technology 2:*

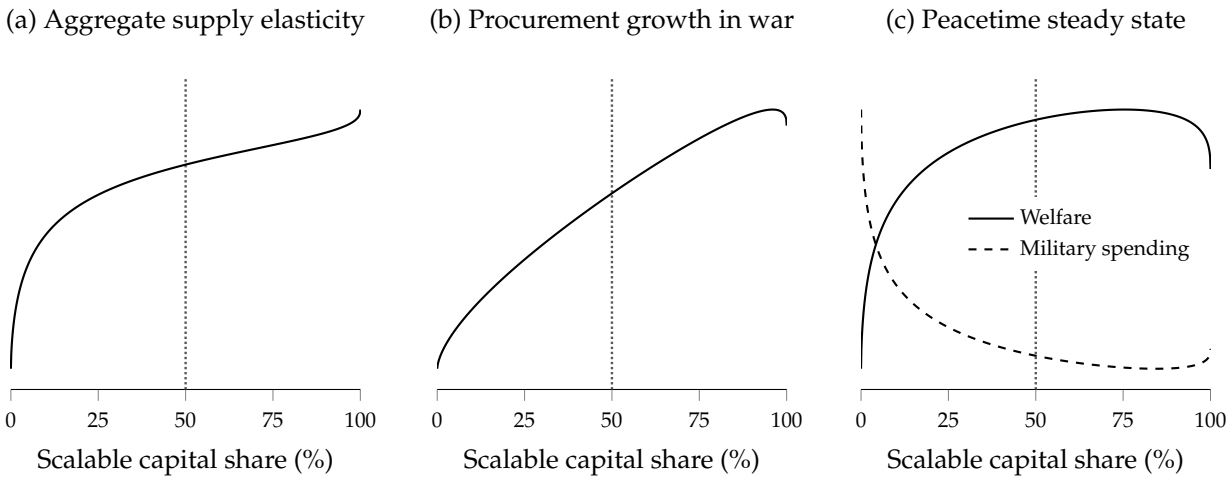
$$s^S_{1,t} > s^S_{2,t}. \tag{33}$$

Proof. Found in Appendix A.5. □

Proposition 6 shows that optimal policy favors more scalable technologies through larger subsidies. Figure 4 illustrates how changing the composition of capital can strengthen deterrence. Starting from a no-subsidy steady state in which both technologies have the same amount of capital, we vary the share allocated to the more scalable technology while

[15] Labor supply curves are unchanged between peace and war, and $w_g(l_g)$ represents the marginal opportunity cost of labor in units of the numeraire. We assume positive numeraire consumption during peace and in the first period of conflict.

Figure 4: Scalability and the composition of industrial capital



Note: The dotted line marks the no-subsidy benchmark. Supply elasticity is evaluated at wartime output; procurement growth is relative to peacetime output. In panel (c), welfare is measured by consumption-equivalent gains and military spending by $p_m^P m^P$, on separate vertical scales.

holding total industrial capital and deterrence fixed. Panel (a) shows that aggregate short-run supply elasticity rises as more capital is allocated to the scalable technology. Panel (b) illustrates how greater scalability can support a larger wartime expansion in procurement.[16] Panel (c) shows that changing the composition of capital alone can raise peacetime steady-state welfare and lower military expenditure: a greater capacity to expand wartime production reduces the peacetime buildup needed to sustain deterrence.

3.5 Extensions

In this section, we discuss a number of extensions to the baseline model. We provide an overview here and delegate details to Appendix B.

Traditional Rational Deterrence Theory Traditional rational deterrence theory has long emphasized the problem of credibility: a country may threaten to fight but prefer to back down once challenged. Our baseline model abstracts from this problem by assuming that Sovereign always fights when Adversary challenges the status quo. But there are

[16]The increase in procurement growth reverses at very high shares of the scalable technology. This reflects two opposing effects of changing the composition of capital. Below the no-subsidy benchmark, increasing the scalable technology's capital share makes supply more elastic and lowers the cost of producing the benchmark peacetime quantity. Above the benchmark, further increases make supply more elastic but raise that cost. Initially, the scalability effect dominates. At very high shares, however, the cost effect dominates: the marginal cost of producing a given wartime quantity rises and wartime output falls. This explains why procurement growth rises monotonically below the benchmark but not above it.

historical examples in which countries have not responded to such challenges with war. For example, France did not respond to Germany's reoccupation of the Rhineland with military force. Similarly, Ukraine in 2014 did not attempt to retake Crimea.

To study how this credibility problem affects optimal industrial policy, we study an extended model in which we explicitly model the possibility of backing down. A country that wants to deter needs to commit to using military force when the status quo is challenged. This implies that countries also need to satisfy a limited commitment constraint. The cost of war needs to be sufficiently low for a country to be able to commit to going to war. This constraint generates another argument for industrial policy. Countries choose capital stocks in order to lower the cost of war. The optimal industrial policy that implements these capital stocks subsidizes capital that is valuable during war. The expression is identical to the one found in Kooi (2025). That paper develops many of its implications in an open-economy general-equilibrium model. It shows that this argument for industrial policy can also support investment in scalable technologies. Because many of the implications are already developed there, we do not develop them again here.

Equilibrium Wars In our baseline model, we study a deterrence equilibrium in which war never occurs. We now add shocks to Adversary's willingness to attack, so that it sometimes chooses war despite Sovereign's deterrence efforts. The deterrence constraint now depends on capital and military stocks in all future periods because Adversary compares attacking today with waiting and retaining the option to attack later. This captures the preventive-war incentive associated with the Thucydides trap: a stronger Sovereign tomorrow can make an attack more attractive to Adversary today. The subsidy formula in Proposition 2 retains its form, but γ_t^S now also reflects how additional capital at date t affects incentives to attack at earlier dates.

Endogenous War Duration Our baseline model assumes that the duration of war is exogenous. We therefore study an extended model in which the probability of reaching the second period depends on the initial military balance, assuming that a larger Sovereign military stock makes a long war more likely. We show that this does not change the optimal subsidy formula in Proposition 2. But it does change the shape of the deterrence function in two ways. First, industrial capital contributes to deterrence only in long wars, so its deterrent value is weighted by the probability of reaching the second period. Second, there is now an additional effect of initial military equipment, namely that it changes the probability that the war lasts into the second period. If long wars are at least as favorable for deterrence as short wars, then in the steady state Sovereign chooses a larger mil-

itary stock relative to capital in the defense industrial base, and therefore chooses lower subsidies than in the case where wars are always long.

Strategic Interactions In our baseline model, Adversary chooses whether to attack, but its military stock is fixed. We extend the model to allow for a more general form of strategic interaction in which Adversary adjusts its military stock in response to Sovereign's peacetime choices. Specifically, Adversary's military stock is allowed to respond separately to Sovereign's military stock and to Sovereign's capital in the defense industrial base. In the steady state, when wars are long, a stronger response to Sovereign's military stock makes that stock less effective at providing deterrence, so Sovereign relies more on the defense industrial base and optimal capital subsidies rise. A stronger response to Sovereign's defense-industrial capital makes that capital less effective at providing deterrence, so optimal capital subsidies fall.

4 Quantitative Model

Our baseline model shows that expected war duration is a key part of the argument for industrial policy. It also shows that such policies should encourage the capacity to expand production. But these points were developed in a simple model in which a single military good was produced by a single sector representing the defense industrial base, and the capacity to scale was modeled only by allowing flexible inputs to adjust. In reality, the defense industrial base and the civilian sector consist of multiple industries, all relying on other industries' output to produce. The capacity to scale is also dynamic because it may depend on the ability to produce and install additional capital goods.

To assess how policy should be targeted across various sectors, we therefore develop the Integrated Defense Economics Model (IDEM), which incorporates all these elements. First, we enrich the military block by explicitly modeling how various military inputs contribute to battlefield outcomes and allowing them to depreciate at different rates in peacetime and wartime. Second, we enrich the economic block with an input-output network to model the capacity to scale production by reallocating flexible inputs such as labor and intermediate goods. Finally, we allow for an investment network to model how industries can scale over time by installing more capital goods.

We present the model below and study optimal policy in the next section. To simplify the exposition, we describe Sovereign's economy, suppressing the country index and dependence on the history of geopolitical states.

4.1 Economic Block

Let $\mathcal{G}$ denote the set of N sectors. All aggregators below have constant returns to scale.

Preferences The representative household consumes sectoral goods $c_{g,t}$, supplies labor $L_{g,t}$ to each sector, and values the public good G_t. The discount factor is β, consumption weights are ϵ_g, and σ_C is the elasticity of substitution across goods in consumption. Following Vom Lehn and Winberry (2022), τ is the elasticity of labor supply across sectors and ξ is the Frisch elasticity of the aggregate labor index. Preferences are

$$\sum_{t=0}^{\infty} \beta^t \Big(\log C_t - \frac{\tilde{L}_t^{1+1/\xi}}{1+1/\xi} + G_t \Big), \quad C_t = \Big[\sum_{g \in \mathcal{G}} \epsilon_g^{\frac{1}{\sigma_C}} \, c_{g,t}^{\frac{\sigma_C - 1}{\sigma_C}} \Big]^{\frac{\sigma_C}{\sigma_C - 1}}, \quad \tilde{L}_t = \Big[\sum_{g \in \mathcal{G}} L_{g,t}^{\frac{\tau+1}{\tau}} \Big]^{\frac{\tau}{\tau+1}}.$$

Production Sector g uses capital $K_{g,t}$, labor $L_{g,t}$, and intermediate goods $x_{g,r,t}$ purchased from sectors r. Let $V_{g,t}$ denote value added, $X_{g,t}$ the intermediate composite, and $Y_{g,t}$ gross output. The α's are sector-specific technology weights. The elasticities σ_{VA}, σ_X, and σ_Y govern substitution between capital and labor, across intermediate goods, and between value added and intermediates, respectively, and are common across sectors:

$$V_{g,t} = \Big[\alpha_{K,g} \, K_{g,t}^{\frac{\sigma_{VA}-1}{\sigma_{VA}}} + \alpha_{L,g} \, L_{g,t}^{\frac{\sigma_{VA}-1}{\sigma_{VA}}} \Big]^{\frac{\sigma_{VA}}{\sigma_{VA}-1}}, \qquad X_{g,t} = \Big[\sum_{r \in \mathcal{G}} \alpha_{g,r} \, x_{g,r,t}^{\frac{\sigma_X - 1}{\sigma_X}} \Big]^{\frac{\sigma_X}{\sigma_X - 1}},$$

$$Y_{g,t} = \Big[\alpha_{V,g} \, V_{g,t}^{\frac{\sigma_Y - 1}{\sigma_Y}} + \alpha_{X,g} \, X_{g,t}^{\frac{\sigma_Y - 1}{\sigma_Y}} \Big]^{\frac{\sigma_Y}{\sigma_Y - 1}}.$$

Capital Accumulation Sector-specific capital depreciates at rate δ_g. Investment is irreversible, $I_{g,t} \geq 0$, and adjustment costs governed by $\varphi > 0$ reduce effective investment, following Vom Lehn and Winberry (2022). The weights $\vartheta_{g,r}$ describe the composition of sector g's investment purchases $i_{g,r,t}$:

$$K_{g,t+1} = (1 - \delta_g) K_{g,t} + I_{g,t} - \frac{\varphi}{2} \Big(\frac{I_{g,t}}{K_{g,t}} - \delta_g \Big)^2 K_{g,t}, \qquad I_{g,t} = \prod_{r \in \mathcal{G}} i_{g,r,t}^{\vartheta_{g,r}}, \qquad g \in \mathcal{G}.$$

Military Power The government accumulates J military stocks $M_{j,t}$, indexed by $j \in \mathcal{J}$. Suppliers are partitioned into sets $\mathcal{G}_j \subset \mathcal{G}$; weights $\psi_{j,g}$ aggregate purchases $m_{j,g,t}$ into non-negative procurement $m_{j,t}$. Depreciation $\delta_{j,t}$ equals δ_j in peace and after war, and $\delta_j^w \geq \delta_j$ during war to reflect combat losses. The weights ϕ_j determine each stock's

contribution to military power M_t:[17]

$$m_{j,t} = \prod_{g \in \mathcal{G}_j} m_{j,g,t}^{\psi_{j,g}}, \qquad M_{j,t+1} = (1 - \delta_{j,t}) M_{j,t} + m_{j,t}, \qquad M_t = \prod_{j \in \mathcal{J}} \Big(\frac{M_{j,t}}{\phi_j} \Big)^{\phi_j}.$$

Market Clearing For each sector, labor supply equals labor demand at the sectoral wage $w_{g,t}$, and sectoral output equals consumption, intermediate, investment, and military demand:

$$Y_{g,t} = c_{g,t} + \sum_{b \in \mathcal{G}} x_{b,g,t} + \sum_{b \in \mathcal{G}} i_{b,g,t} + \sum_{j \in \mathcal{J}} m_{j,g,t}, \qquad g \in \mathcal{G}.$$

Competitive Equilibrium The competitive equilibrium is analogous to that in Section 2.1. The key change is that subsidies apply at rate $s_{g,t}$ to capital's replacement value $P^I_{g,t} K_{g,t}$, where $P^I_{g,t}$ is the price of sector g's investment basket. The per-unit subsidy of the baseline model corresponds to $s_{g,t} P^I_{g,t}$. Let $R_t = \prod_{v=0}^{t-1} (1 + r_v)$, with $R_0 = 1$, denote the cumulative gross return. With goods prices $p_{g,t}$ and wages $w_{g,t}$, the producer's objective now reads

$$\sum_{t=0}^{\infty} \frac{1}{R_t} \Big(p_{g,t} Y_{g,t} + s_{g,t} P^I_{g,t} K_{g,t} - w_{g,t} L_{g,t} - \sum_{r \in \mathcal{G}} p_{r,t} \big(x_{g,r,t} + i_{g,r,t} \big) \Big).$$

Appendix C.1 derives the optimality conditions, and defines the competitive equilibrium.

4.2 Deterrence Block

Timing and War Duration Military outcomes in the initial period determine whether a war is short or long. If Sovereign avoids a quick defeat, the conflict becomes an "industrial war of attrition," whose duration follows an exogenous distribution. Let π_T denote baseline probabilities of wars lasting $T \in \{1, \ldots, \bar{T}\}$ periods. The initial balance of power scales the probability of a long war by $a(M_t)$: a stronger Adversary is more likely to defeat Sovereign quickly. With M_0^A denoting Adversary's initial military power, we parameterize

$$\tilde{\pi}_T(M_t) = a(M_t)\, \pi_T \quad (T \geq 2), \qquad \tilde{\pi}_1(M_t) = 1 - a(M_t)\, (1 - \pi_1),$$
$$a(M_t) = 1 - \frac{\kappa}{1 - \pi_1} \Big(\frac{M_0^A}{M_t + M_0^A} - \frac{1}{2} \Big).$$

[17] We collect military stocks in $\mathbf{M}_t$ and sectoral capital stocks in $\mathbf{K}_t$.

At parity ($M_t = M_0^A$) the probabilities are the baseline frequencies π_T; the probability of a short war rises by κ percentage points per percentage point of Adversary's share of military power. The probability of reaching the attrition phase is therefore $a(M_t)(1-\pi_1)$; conditional on reaching it, duration follows the fixed distribution $\pi_T/(1-\pi_1)$ for $T \geq 2$.[18]

The Allocation of the Public Good Re-index dates so that war begins at date zero. The parameters $\chi_s^{(T)}$ measure the effectiveness of military power in period s of a war lasting T periods; M_s^A denotes Adversary's military power. The postwar settlement changes the public good from its initial level G_0 according to

$$G_t = G_0 + \sum_{s=0}^{T-1} \chi_s^{(T)} \left(M_s - M_s^A\right) \quad \text{if } t \geq T, \qquad \text{and} \qquad G_t = G_0 \quad \text{otherwise.}$$

Adversary's Policy and Challenge Decision Adversary's military power is constant in peace and follows an exogenous path $\{M_s^A\}_{s=0}^{T-1}$ in a war of length T. Mobilization is costly for Adversary as well: from the outbreak of a war of length T, its consumption and labor supply follow exogenous paths that depart from their peacetime levels (see Appendix C.2 for details). Adversary attacks when $W_t^{A,C} + \eta_t > W_t^{A,P}$, where η_t is its political preference for war. It decides before the duration is known, so welfare from war $W_t^{A,C}$ is the expectation of $W_t^{A,C,(T)}$ across durations of war T:

$$W_t^{A,C} = \sum_{T=1}^{\bar{T}} \tilde{\pi}_T(M_t)\, W_t^{A,C,(T)}.$$

Sovereign's Wartime and Postwar Policy As in the baseline model, Sovereign optimally chooses wartime military expenditure, trading off consumption against military gains in the settlement (see Appendix C.2 for details on Sovereign's wartime problem). After war, peace is permanent and the public good is fixed. The departure from the baseline model is that postwar expenditure follows an exogenous rule: the government spends a share θ of GDP on procurement, allocated across baskets with shares θ_j. This rule allows the model to capture a realistic postwar reduction in military procurement. Under the rule, the military stocks carried out of a war have no value to Sovereign, so purchases in the last period of the war, which arrive only after the settlement, do not affect it; they follow the postwar rule. Industrial policy is not needed during or after the war.

[18] We represent this distinction in reduced form: the duration is drawn and revealed to both parties at the outbreak.

The Simplified Deterrence Constraint Let $M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t)$ be Sovereign's optimal military power in period s of a war lasting T periods, starting from date-t stocks $(\mathbf{K}_t, \mathbf{M}_t)$. Let $\mathcal{E}^{(T)}$ denote the present value, at the outbreak, of the utility Adversary loses in war relative to peace, due to lower consumption and higher labor disutility. Sovereign's stocks affect Adversary's economic cost only through the war-length probabilities: a militarily stronger Sovereign makes a long war, which is costlier for Adversary, more likely. Deterrence requires the expected discounted settlement plus this economic cost to offset Adversary's preference for war (see Appendix C.2 for details on deriving the simplified deterrence constraint):

$$\mathcal{D}_t(\mathbf{K}_t, \mathbf{M}_t) = \sum_{T=1}^{\bar{T}} \tilde{\pi}_T(M_t) \left[\frac{\beta^T}{1-\beta} \sum_{s=0}^{T-1} \chi_s^{(T)} \left(M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t) - M_s^A \right) + \mathcal{E}^{(T)} \right] \geq \eta_t, \qquad \forall t. \tag{34}$$

As in Definition 2, a deterrence equilibrium in this model is a competitive equilibrium satisfying the deterrence constraint (34) in every period.

4.3 Calibration

We calibrate the model to the United States, at annual frequency. The economic structure and military stocks are disciplined by sectoral accounts and defense procurement data; war durations and wartime spending discipline the conflict parameters. The main text gives an overview of our calibration procedure. Appendix C.3 provides additional details.

4.3.1 Economic Block

We set the number of sectors to $N = 370$ (see Appendix C.3 on how we choose these sectors). We calibrate elasticities and the adjustment-cost parameter based on the existing literature. Table 1 reports these economic parameters and their sources. Given these parameters we set our consumption weights and the sector-specific technology weights so that the resulting spending shares match the 2017 BEA input-output accounts. Investment weights $\vartheta_{g,r}$ come from Iovino et al. (2025), who extend the investment network of Vom Lehn and Winberry (2022) to the 2017 BEA detail level, and the sectoral depreciation rates δ_g are computed from the BEA fixed-asset tables. Appendix C.3 explains the calibration of the consumption and technology weights and the sectoral depreciation rates in more detail.

Table 1: Economic parameters.

Parameter	Value	Source
β, discount factor	0.96	Vom Lehn and Winberry (2022)
ξ, aggregate Frisch elasticity	0.4	Reichling and Whalen (2012)
τ, sectoral labor supply elasticity	4.5	Vom Lehn and Winberry (2022)
φ, capital adjustment cost	0.5	Vom Lehn and Winberry (2022)
σ_Y, value added–intermediates elasticity	0.5	Baqaee and Farhi (2019)
σ_{VA}, capital–labor elasticity	0.5	Oberfield and Raval (2021)
σ_X, intermediate-input elasticity	0.001	Baqaee and Farhi (2019)
σ_C, consumption-goods elasticity	0.9	Baqaee and Farhi (2019)

4.3.2 Deterrence Block

The deterrence block is the novel part of IDEM, and so we discuss its calibration in some detail. The question is how one calibrates a model in which wars do not occur on the equilibrium path. Our approach is to build on the literature in macroeconomics. It is common to calibrate models to match steady-state moments but also to match impulse responses to shocks that never happen on the equilibrium path—so-called MIT shocks. We will follow this approach, but there is one important difference. In standard models, the steady state can often be calibrated without specifying the MIT shocks—since they do not happen in equilibrium, they do not affect the steady state. That is not true in our model because what the war looks like does determine what is needed to deter it, even if it is avoided on the equilibrium path. As a result, for some parameters, the steady state and wartime impulse response have to be calibrated jointly.

We are interested in studying the gains from industrial policy. We therefore calibrate our model on a baseline steady state in which the government does not directly manipulate industrial capacity to deter war. In this no-industrial-policy steady state, it buys military equipment by looking at its contribution to deterrence but does not take into account that its purchases change the allocation of capital in equilibrium. Appendix C.4 describes this benchmark no-industrial-policy steady state in more detail and presents the optimality conditions.

Military Baskets We calibrate $J = 8$ baskets of military equipment: structures, other equipment, intellectual property, non-durables, land systems, air systems, naval systems, ammunition. Appendix C.3 describes the composition of these baskets in more detail. We calibrate aggregate postwar military spending on these baskets to $\theta = 3.37\%$ of GDP, the US average over 2015–2025 in the SIPRI data. We calibrate the postwar expenditure

shares θ_j to match 2017 US defense procurement. The within-basket Cobb-Douglas share of sector g, $\psi_{j,g}$, is similarly calibrated on the within-basket expenditure shares in 2017 US defense procurement data. Table 7 in Appendix C.3 reports these shares.

Depreciation of Military Equipment Peacetime depreciation rates for each equipment type come from BEA national-defense fixed assets, with ammunition depreciation measured by purchases relative to stocks in DoD inventory reports. Wartime rates add combat losses from the Russia–Ukraine war for land systems and US World War II experience for air and naval systems. Structures, other equipment, and intellectual property retain their peacetime rates. We think of this as the case when a war is fought offshore rather than on a country's own territory. Non-durables are consumed within the year and depreciate fully. Munitions also depreciate fully; this reflects the idea that they can be stockpiled, but they are fully expended when used in combat. Table 5 shows the calibrated depreciation rates, and Appendix C.3 provides more details on the datasets and the procedure.

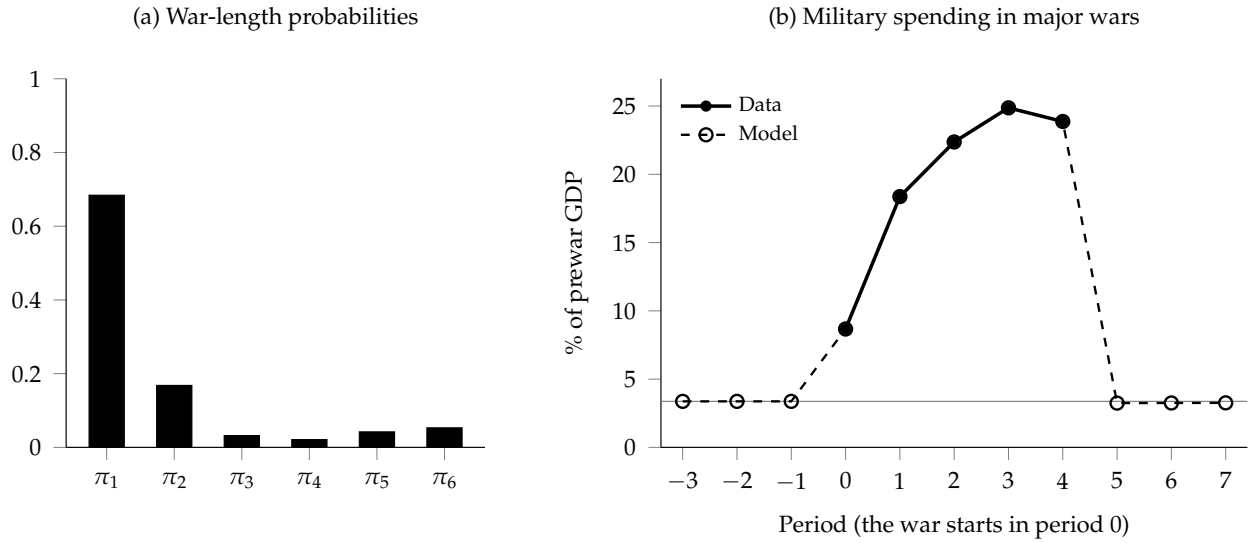
War Duration We calibrate our distribution of war duration to match inter-state wars with at least 1,000 battle deaths in the Correlates of War data, 1816–2007. We first calibrate $\kappa = 0.33$ to match the fact that in the National Material Capabilities data a one-percentage-point rise in the initiator's prewar share of military personnel is associated with a reduction in the probability of a war longer than one year by 0.33 percentage points. Conditional on κ, we calibrate baseline probabilities to match annual duration frequencies of inter-state wars. Panel (a) of Figure 5 plots the distribution. Wars lasting six years or more are pooled at $\bar{T} = 6$. Appendix C.3 provides additional details.

Wartime Military Expenditures The incentive to spend during wartime depends on the value of military equipment on the battlefield. We therefore calibrate combat efficiency parameters $\chi_s^{(T)}$ so that it is optimal for the government to choose spending paths that coincide with those during "major war booms" estimated by Marzian and Trebesch (2026). A war of length T matches the first $T-1$ targets, since our time-to-build assumption implies that equipment arrives with a one-period delay.[19] Panel (b) of Figure 5 shows the impulse response under our calibration for a $T = 6$ war.

[19]The parameter $\chi_0^{(T)}$ cannot be identified directly from the expenditure data because any military equipment procured at this time affects the battlefield in the next period. We therefore compute the present value of additional military equipment at each war date after the first. We find it essentially linear and use it to project the date 0 present value of military equipment and calibrate $\chi_0^{(T)}$ to match the projection. We provide details in Appendix C.3 and show that our quantitative results on the sectoral ranking of optimal subsidies are robust to other calibrations of $\chi_0^{(T)}$ in Appendix C.7.1.

Peacetime Military Expenditures We calibrate ϕ_j by matching the basket composition of military procurement in 2017 in our peacetime no-industrial-policy steady state. Because the composition of military expenditure depends on its contribution to deterrence, these parameters have to be jointly calibrated with the $\chi_s^{(T)}$ parameters, which govern the wartime spending path. The last parameter we calibrate is η_t. This parameter relaxes or tightens the deterrence constraint and is calibrated so that aggregate military expenditure in the no-industrial-policy peacetime steady state matches the US level of 3.37% of GDP. For a detailed explanation of the calibration of the $\chi^{(T)}$, ϕ, and η_t parameters, see Appendix C.3.

Figure 5: Calibrating the military block.



Note: Panel (a) shows the share of inter-state wars with given lengths in the Correlates of War dataset (π_6 is the share of wars 6 years long or longer). Panel (b) shows military spending as a share of prewar GDP in the major war booms estimated by Marzian and Trebesch (2026) (Data) and in the model's six-year war (Model); the thin line marks the prewar military spending share of GDP of 3.37%.

5 Quantitative Results

In this section, we first briefly describe Sovereign's peacetime planning problem, and the formula for optimal subsidies that the government uses to decentralize the planner's allocation and achieve the optimal deterrence equilibrium. We then present the numerical results on the optimal deterrence equilibrium in our model.

5.1 Optimal Deterrence Equilibrium

Sovereign's peacetime planning problem generalizes that in Section 3.1. We explain how we solve this numerically in Appendix C.6. Given initial stocks, the planner solves

$$\begin{aligned} &\max_{\{c_{g,t},L_{g,t},x_{g,r,t},i_{g,r,t},I_{g,t},K_{g,t+1},m_{j,g,t},m_{j,t},M_{j,t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty}\beta^t\left(\log C_t - \frac{\tilde{L}_t^{1+1/\xi}}{1+1/\xi} + G_0\right) \\ &\text{s.t. (i) the technology and market-clearing conditions (120), for all } t, \\ &\quad \text{(ii) the laws of motion of capital and military stocks (116) and (119), for all } t, \\ &\quad \text{(iii) the deterrence constraint } \mathcal{D}_t(\mathbf{K}_t,\mathbf{M}_t) \geq \eta_t, \text{ for all } t \geq 1, \\ &\quad \text{(iv) the non-negativity constraints } I_{g,t} \geq 0 \text{ and } m_{j,t} \geq 0, \text{ for all } t. \end{aligned} \tag{35}$$

Proposition 2 likewise generalizes to this economy. Let $\gamma_t \geq 0$ denote the shadow value of relaxing the date-t deterrence constraint, expressed in units of date-t consumption.

Proposition 7. *An optimal deterrence equilibrium can be implemented with sectoral capital subsidy rates*

$$s_{g,t} = \frac{\gamma_t}{P_{g,t}^I}\frac{\partial \mathcal{D}_t}{\partial K_{g,t}}, \qquad g \in \mathcal{G}.$$

Proof. See Appendix C.5.[20] □

The subsidy compensates firms for capital's marginal deterrence value, which they do not internalize. Dividing by the investment-basket price $P_{g,t}^I$ converts this value into a rate on capital's replacement value. Capital can affect deterrence through military production and through supplies of intermediate and investment goods to other sectors; the derivative captures these effects through the wartime equilibrium. When $\gamma_t > 0$, positive marginal deterrence effects warrant subsidies and negative effects warrant taxes.

5.2 Aggregate Outcomes

Our model studies how countries can deter efficiently. We are interested in how countries can use industrial policy to lower the cost of sustaining peace. Table 2 shows how industrial policy achieves this in our model. Both steady states provide the same level of deterrence. By complementing peacetime military strength with industrial capacity, the government attains this level with a lower military procurement share of GDP. Relative

[20] The proof of Proposition 7 assumes that $\frac{\partial \mathcal{D}_t}{\partial K_{g,t}}$ exists for all g, t. In practice, we can show numerically that kinks in the deterrence function do not affect our quantitative results.

to the benchmark of deterrence through procurement alone, steady-state welfare rises by 2.58% in consumption-equivalent terms.[21]

Table 2: Aggregate outcomes in the steady state.

$\mathcal{D}$	0.00
welfare	2.58
consumption	3.61
military procurement share	−28.20
real GDP	4.49
aggregate capital	9.01
labor supply	1.13

Note: All entries are percent deviations from the no-industrial-policy benchmark. $\mathcal{D}$ is the value of the deterrence function, at its common binding level $\mathcal{D} = \eta$ in both steady states. Welfare is expressed as the equivalent permanent increase in benchmark consumption. The military procurement share is procurement as a share of GDP, 3.37% in the benchmark. GDP and capital are Fisher quantity indices, and labor supply is the sum of sectoral labor.

The table also explains why steady-state welfare rises. The main driver is a 3.6% increase in consumption, supported by a lower military procurement burden and an increase in steady-state real GDP. Military procurement's share of GDP falls from 3.37% to 2.42%, while real GDP rises by 4.5%. The increase in output is supported by a 9.0% rise in the aggregate capital stock. As discussed in the next subsection, most sectors receive positive subsidies, which encourage investment and capital accumulation. A modest 1.1% increase in labor supply also contributes to production. This increase in labor offsets part of the consumption benefit, explaining why the welfare gain is smaller than the increase in consumption.

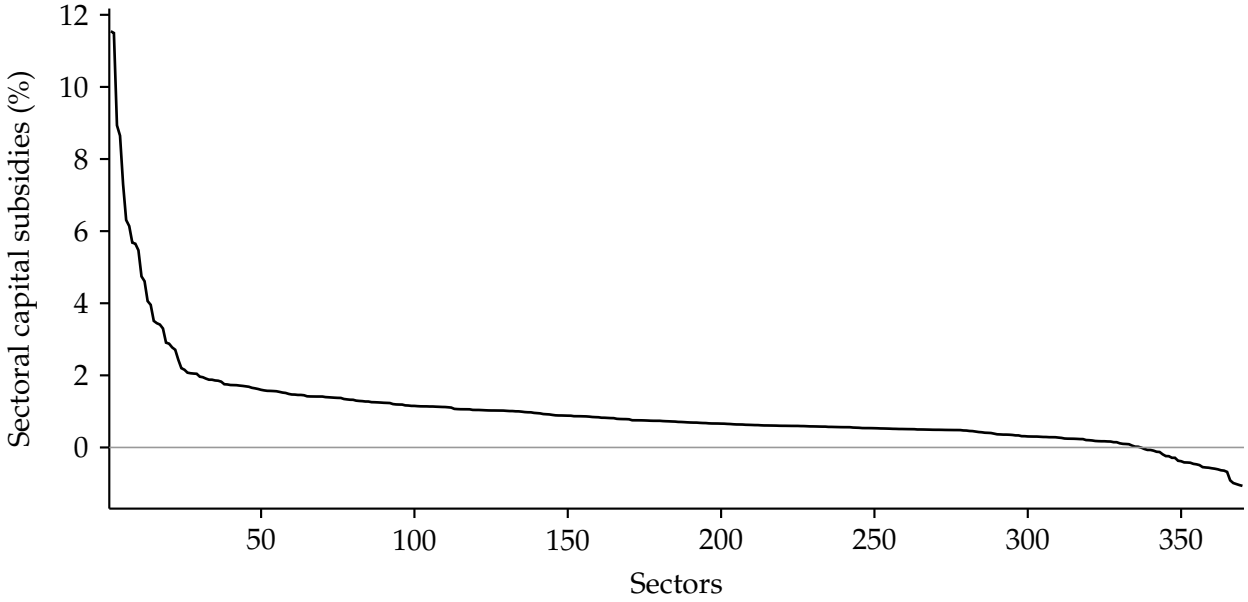
5.3 Industrial Priorities in the Baseline

We now use the quantitative model to study which industries a government should encourage and how much their subsidies should differ. We begin by examining the cross-section of industries. Figure 6 provides a first insight: optimal subsidy rates decline rapidly as one moves down the sectoral ranking. This result is interesting because there is a soft consensus among economists that the national security argument for industrial

[21] Eliminating the deterrence constraint altogether would raise welfare by 4.65% relative to the same benchmark. Industrial policy therefore eliminates approximately half of the steady-state welfare cost of deterrence.

policy at best applies to a narrow range of industries. Figure 6 shows the model delivers this quantitatively: optimal subsidies decline by 75% within the first twenty sectors, falling from 11.54% to 2.88%.

Figure 6: The cross-section of optimal capital subsidies.



Note: The line traces the subsidy rates of the 370 sectors, ordered by decreasing subsidy. The subsidy s_g is a rate on the replacement value of the sector's capital stock.

Next, we present the ranking of industrial priorities. Table 3 reports the twenty industries with the highest subsidy rates; the full ranking appears in Appendix C.7.2. Three broad categories dominate the ranking. First are producers of munitions and their components, including guided missiles, propulsion units, and ammunition. Second are manufacturers of heavy military equipment—ships, aircraft, and armored vehicles—and their input suppliers. Third are producers of advanced electronics, including search, detection, and navigation instruments, electricity and signal testing instruments, and communications equipment.

What determines whether a sector is an industrial priority? Our model highlights two channels. First, military stocks differ in how quickly they are depleted in war, and high wartime depletion raises the value of the capacity to replenish a stock, especially when its losses rise sharply relative to peacetime. A shell can be fired only once, so munitions are replenished more often than tanks, which in turn suffer battlefield losses that military structures do not. The capacity to produce munitions and equipment with high wartime loss rates is therefore particularly valuable.

Table 3: The 20 sectors with the highest optimal capital subsidies.

rank	sector	s_g (%)
1	Guided missile and space vehicle manufacturing	11.54
2	Propulsion units and parts for space vehicles and guided missiles	11.50
3	Ship building and repairing	8.93
4	Search, detection, and navigation instruments manufacturing	8.65
5	Military armored vehicle, tank, and tank component manufacturing	7.30
6	Ammunition, arms, ordnance, and accessories manufacturing	6.31
7	Other aircraft parts and auxiliary equipment manufacturing	6.14
8	Aircraft manufacturing	5.68
9	Truck transportation	5.65
10	Architectural, engineering, and related services	5.47
11	Electronic and precision equipment repair and maintenance	4.74
12	Electricity and signal testing instruments manufacturing	4.61
13	Other computer related services, including facilities management	4.06
14	Broadcast and wireless communications equipment	3.96
15	Water transportation	3.51
16	Switchgear and switchboard apparatus manufacturing	3.45
17	Computer systems design services	3.41
18	Air transportation	3.30
19	Aircraft engine and engine parts manufacturing	2.91
20	Other electronic component manufacturing	2.88

Second, a sector can become an industrial priority through its exposure to military demand along the input-output and investment networks. In Appendix C.7.3, we construct a measure of military centrality which quantifies how important individual sectors are for producing an additional unit of military power through the production and investment networks. We find that sectors with higher military centralities, especially at the top of the centrality distribution, receive markedly higher subsidies on average (see Figure 9).

We also show that expanding capacity in component suppliers can lower the cost of military equipment and allow a larger wartime buildup. In Appendix C.7.4, we show numerically how this mechanism operates across sectors: in the no-industrial-policy steady state, we separately raise the capital stock of each of the twenty sectors in Table 3 to its level in the optimal peacetime steady state. Then, we look at the effect of each of these changes on the optimal allocations in a six-year war. In propulsion unit manufacturing, additional capacity supports direct military purchases of ammunition and lowers the cost of inputs used in missile production. Suppliers of investment goods, such as switchgear or communications equipment, also help expand production capacity during war.

5.4 Industrial Priorities and the Nature of Conflict

Optimal industrial policy depends not only on economic fundamentals, such as the input-output and investment networks, but also on the war a country expects to fight. Different conflicts require different mixes of military equipment, giving rise to different industrial priorities. When wars may be prolonged, economic policymaking and military analysis therefore need to be closely linked. To illustrate this point, we use the model to consider three stylized scenarios: a land war, an air war, and a naval war. In each scenario, we set the military-power weights of the other two platform categories to zero, rescale the remaining weights proportionally so that they still sum to one, and solve again for the optimal deterrence equilibrium. The combat efficiency parameters and all other parameters keep their baseline values, except that, as in the baseline, Adversary's wars mirror Sovereign's and η is set so that the deterrence constraint binds at the no-industrial-policy steady state. Common military inputs, including ammunition, continue to contribute to military power in all three scenarios.

Table 4 reports baseline subsidy rates and the changes under each scenario for five industries whose subsidies vary substantially across scenarios. Shipbuilding receives a subsidy of 8.93% in the baseline, which rises to 8.98% in the naval scenario but falls to around 1% in the land and air scenarios. The subsidy to armored vehicles and tanks is almost unchanged in the land scenario and falls by roughly four percentage points in the other two. Subsidies to aircraft and their components remain close to their baseline levels in the air scenario but fall substantially in the land and naval scenarios.

Table 4: Changes in optimal capital subsidies across conflict scenarios.

	Baseline	Change from baseline (pp)		
Sector	(%)	Land	Air	Naval
Shipbuilding	8.93	−8.12	−7.82	+0.05
Armored vehicles, tanks, and components	7.30	−0.21	−4.14	−4.50
Other aircraft parts and auxiliary equipment	6.14	−5.15	−0.07	−5.12
Aircraft manufacturing	5.68	−4.59	−0.07	−4.55
Aircraft engines and parts	2.91	−1.84	+0.09	−1.83

Note: Baseline entries are subsidy rates s_g in percent; scenario entries are changes in percentage points. Sector names are abbreviated. Appendix C.7.5 reports subsidy levels for all twenty industries in Table 3.

Other industries remain priorities across all three scenarios. As the fuller comparison in Appendix C.7.5 shows, guided missile and propulsion unit manufacturing retain subsidy rates around 11%, while ammunition and search, detection, and navigation instruments also remain highly subsidized. These industries supply military inputs that

remain important across the scenarios.

6 Conclusion

This paper develops a framework to study the role of economic policy in the deterrence of long industrial wars. When wars are short, there is no role for policy to support investment in industrial capacity. The framework therefore explains why early work on defense economics tended to not focus on industrial capacity. If a war is expected to end quickly, then the focus of policy should be on the strategic placement of troops and weapons so that they can be deployed quickly and second-strike capacity, in the case of nuclear conflicts, can be maintained. The early defense economists allocated their scarce time accordingly. But when wars are long, there is a much greater role for economic policy because industrial capacity can act as a military deterrent.

The return of long industrial wars implies a greater integration between economic analysis and military science is needed. After all, one has to decide in which industries production capacity should be supported. We therefore developed an IDEM—integrating a military block, an economic block and a description of the aggression decision—to aid this analysis. Our IDEM is a first step in this direction, but there is much more that could be done. Contemporaneous work by Grassi and Markó (2026) integrating combat models that generate Lanchester laws with general equilibrium models of production could further enrich IDEM's military block. We also did not model geography. Some countries with large landmasses have the advantage of strategic depth, allowing for retreats without loss of all industrial capacity. Industrial capacity can be disrupted not just through conquest but also through air power. Our current model allows for airpower, but it's closest to its tactical role in supporting other armed forces on the front line, while a spatial model could also capture its strategic role in disrupting an adversary's economy. We intend to incorporate these considerations into the next generation of IDEM in future work.

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A Proofs

A.1 Derivation of (25) and (26)

Proof. We derive the planner optimality conditions at an interior peacetime allocation, allowing the wartime labor constraint to bind. We use two smooth functions, $\mathcal{D}_t^I$ and $\mathcal{D}_t^F$, where I denotes incentives and F denotes feasibility.

For short wars ($T = 1$), wartime production cannot affect the military outcome, so we define both functions identically:

$$\mathcal{D}_t^I(K, M) = \mathcal{D}_t^F(K, M) = \frac{\chi}{1-\beta}(M - \bar{M}^A).$$

For long wars ($T = 2$), let $B \equiv \chi\beta/(1-\beta)$ and $a \equiv [B(1-\alpha)]^{(1-\alpha)/\alpha}$, and define

$$\begin{aligned}\mathcal{D}_t^I(K, M) &= B\big[(2-\delta_M)M + aK - 2\bar{M}^A\big],\\ \mathcal{D}_t^F(K, M) &= B\big[(2-\delta_M)M + K^\alpha(\bar{l}^S)^{1-\alpha} - 2\bar{M}^A\big].\end{aligned} \tag{36}$$

Here $\mathcal{D}_t^I$ uses the wartime production Sovereign would choose in the absence of the labor constraint, while $\mathcal{D}_t^F$ uses the maximum output attainable with full labor mobilization. For either war length, the deterrence function is

$$\mathcal{D}_t(K, M) = \min\left\{\mathcal{D}_t^I(K, M), \mathcal{D}_t^F(K, M)\right\}.$$

Thus $\mathcal{D}_t \geq \eta_t^A$ is equivalent to imposing both $\mathcal{D}_t^I \geq \eta_t^A$ and $\mathcal{D}_t^F \geq \eta_t^A$. Throughout the derivation, $j \in \{I, F\}$. This formulation covers differentiable allocations and the kink using ordinary multipliers.

Suppress the argument $z_t = P$ and write the constraints as

$$c_t^S + i_t^S = l_{n,t}^S, \tag{37}$$
$$K_{t+1}^S = (1-\delta_K)K_t^S + i_t^S, \tag{38}$$
$$l_{n,t}^S + l_{m,t}^S = \bar{l}^S, \tag{39}$$
$$m_t^S = (K_t^S)^\alpha (l_{m,t}^S)^{1-\alpha}, \tag{40}$$
$$M_{t+1}^S = (1-\delta_M)M_t^S + m_t^S, \tag{41}$$
$$\mathcal{D}_{t+1}^j(K_{t+1}^S, M_{t+1}^S) \geq \eta_{t+1}^A, \qquad j \in \{I, F\}. \tag{42}$$

Let $\lambda_{n,t}^S$ be the multiplier on (37), $\lambda_{K,t}^S$ on (38), $\lambda_{l,t}^S$ on (39), $\lambda_{y,t}^S$ on (40), $\lambda_{m,t}^S$ on (41), and $\gamma_{j,t+1}^S$ on branch j of (42). Define $\gamma_t^S \equiv \sum_j \gamma_{j,t}^S$. For short wars, the two constraints coincide, so only the sum of their multipliers matters. The deterrence multipliers satisfy, for $t \geq 1$,

$$\gamma_{j,t}^S \geq 0, \qquad \gamma_{j,t}^S\big[\mathcal{D}_t^j(K_t^S, M_t^S) - \eta_t^A\big] = 0, \qquad j \in \{I, F\}.$$

The Lagrangian is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big(& c_t^S + \lambda_{n,t}^S (l_{n,t}^S - c_t^S - i_t^S) + \lambda_{K,t}^S ((1-\delta_K) K_t^S + i_t^S - K_{t+1}^S) \\ & + \lambda_{l,t}^S (\bar{l}^S - l_{n,t}^S - l_{m,t}^S) + \lambda_{y,t}^S ((K_t^S)^\alpha (l_{m,t}^S)^{1-\alpha} - m_t^S) \\ & + \lambda_{m,t}^S ((1-\delta_M) M_t^S + m_t^S - M_{t+1}^S) \\ & + \beta \sum_j \gamma_{j,t+1}^S (\mathcal{D}_{t+1}^j (K_{t+1}^S, M_{t+1}^S) - \eta_{t+1}^A) \Big).\end{aligned}$$

First-Order Conditions (within Period). For each t, the FOCs w.r.t. $c_t^S, i_t^S, l_{n,t}^S, l_{m,t}^S, m_t^S$ are

$$[c_t^S]: \quad 1 - \lambda_{n,t}^S = 0, \tag{43}$$

$$[i_t^S]: \quad -\lambda_{n,t}^S + \lambda_{K,t}^S = 0, \tag{44}$$

$$[l_{n,t}^S]: \quad \lambda_{n,t}^S - \lambda_{l,t}^S = 0, \tag{45}$$

$$[l_{m,t}^S]: \quad -\lambda_{l,t}^S + \lambda_{y,t}^S (1-\alpha)(K_t^S)^\alpha (l_{m,t}^S)^{-\alpha} = 0, \tag{46}$$

$$[m_t^S]: \quad -\lambda_{y,t}^S + \lambda_{m,t}^S = 0. \tag{47}$$

First-Order Conditions for the Stocks M_{t+1}^S and K_{t+1}^S. Differentiating $\mathcal{L}$ w.r.t. M_{t+1}^S yields

$$[M_{t+1}^S]: \quad -\beta^t \lambda_{m,t}^S + \beta^{t+1} (1-\delta_M) \lambda_{m,t+1}^S + \beta^{t+1} \sum_j \gamma_{j,t+1}^S \frac{\partial \mathcal{D}_{t+1}^j}{\partial M_{t+1}^S} = 0. \tag{48}$$

Differentiating $\mathcal{L}$ w.r.t. K_{t+1}^S yields

$$\begin{aligned}[K_{t+1}^S]: \quad & -\beta^t \lambda_{K,t}^S + \beta^{t+1} (1-\delta_K) \lambda_{K,t+1}^S \\ & + \beta^{t+1} \lambda_{y,t+1}^S \alpha (K_{t+1}^S)^{\alpha-1} (l_{m,t+1}^S)^{1-\alpha} + \beta^{t+1} \sum_j \gamma_{j,t+1}^S \frac{\partial \mathcal{D}_{t+1}^j}{\partial K_{t+1}^S} = 0.\end{aligned} \tag{49}$$

Optimality Conditions and Differentiable Allocations. Equations (43)–(44) imply $\lambda_{n,t}^S = \lambda_{K,t}^S = 1$, and (47) gives $\lambda_{y,t}^S = \lambda_{m,t}^S$. Every branch has the same derivative with respect to M, which equals $\partial \mathcal{D}_t / \partial M$ even at the kink. Dividing (48) by β^t therefore gives

$$\lambda_{m,t}^S = \beta \Big(\gamma_{t+1}^S \frac{\partial \mathcal{D}_{t+1}}{\partial M_{t+1}^S} + (1-\delta_M) \lambda_{m,t+1}^S \Big),$$

which is (25).

Similarly, (49) becomes

$$1 = \beta\Big(\sum_j \gamma^S_{j,t+1} \frac{\partial \mathcal{D}^j_{t+1}}{\partial K^S_{t+1}} + (1-\delta_K) + \lambda^S_{m,t+1} \alpha (K^S_{t+1})^{\alpha-1} (l^S_{m,t+1})^{1-\alpha} \Big). \tag{50}$$

For short wars, both branches have zero derivative with respect to capital. For long wars away from the kink, only the branch attaining the minimum can have a positive multiplier. In either case, the sum in (50) reduces to $\gamma^S_{t+1}\, \partial \mathcal{D}_{t+1} / \partial K^S_{t+1}$, recovering (26). At the long-war kink, both branches can carry positive multipliers and (50) continues to apply. The main-text derivative formulas are thus the differentiable special cases of these conditions. □

A.2 Proof of Proposition 1

Fix a possible challenge date t. The deterrence constraint compares peace with the off-equilibrium continuation in which Adversary challenges at t. By (22), deterrence is

$$\mathcal{D}_t(K^S_t, M^S_t) = \frac{\chi \beta^{T-1}}{1-\beta} \sum_{\tau=0}^{T-1} \left(M^{S,C}_\tau (K^S_t, M^S_t) - \bar{M}^A \right).$$

Short Wars. Suppose first that $T = 1$. Then only the military stock available at the possible challenge date matters for the outcome:

$$\mathcal{D}_t(K^S_t, M^S_t) = \frac{\chi}{1-\beta} \left(M^S_t - \bar{M}^A \right).$$

Military production in that continuation affects $M^{S,C}_1$, but this stock is reached only after a short war would have ended. It therefore has no effect on the allocation of the public good. Hence

$$\frac{\partial \mathcal{D}_t}{\partial M^S_t} = \frac{\chi}{1-\beta} > 0, \qquad \frac{\partial \mathcal{D}_t}{\partial K^S_t} = 0.$$

This gives the first part of the proposition.

Long Wars. Suppose now that $T = 2$, and let $B \equiv \chi\beta/(1-\beta)$ denote the marginal value of first-period conflict production in a long war. The deterrence function is

$$\mathcal{D}_t(K^S_t, M^S_t) = B \left(M^S_t + M^{S,C}_1 - 2\bar{M}^A \right).$$

In this off-equilibrium continuation, the date-t stocks are predetermined, so

$$M_1^{S,C} = (1-\delta_M)M_t^S + m_0^{S,C}.$$

The wartime choice of military production depends on K_t^S, but not on M_t^S, because M_t^S enters the wartime objective additively and separably. Therefore

$$\frac{\partial \mathcal{D}_t}{\partial M_t^S} = B\Big(1 + (1-\delta_M)\Big) > 0.$$

It remains to characterize the derivative with respect to the defense-industrial capital stock. In the first period of the off-equilibrium long-war continuation, capital is predetermined and Sovereign chooses labor in the defense sector. Using the resource constraint to substitute out consumption, and noting that from the final period of the war all labor is allocated to producing consumption goods, the relevant static problem is

$$\max_{0 \le l_{m,0}^{S,C} \le \bar{l}^S} \left\{ -l_{m,0}^{S,C} + B(K_t^S)^\alpha \big(l_{m,0}^{S,C}\big)^{1-\alpha} \right\}.$$

Some Labor Remains in Numeraire Production. If some labor remains in numeraire production in the first period of war, the choice of $l_{m,0}^{S,C}$ is interior. The first-order condition is

$$B(1-\alpha)(K_t^S)^\alpha \big(l_{m,0}^{S,C}\big)^{-\alpha} = 1,$$

so

$$l_{m,0}^{S,C}(K_t^S) = \big(B(1-\alpha)\big)^{1/\alpha} K_t^S.$$

Substituting this policy into the production function gives

$$m_0^{S,C}(K_t^S) = \big(B(1-\alpha)\big)^{\frac{1-\alpha}{\alpha}} K_t^S.$$

It follows that

$$\frac{\partial \mathcal{D}_t}{\partial K_t^S} = B\frac{\partial m_0^{S,C}}{\partial K_t^S} = \left(\frac{\chi\beta}{1-\beta}\right)^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} > 0.$$

All Labor Is Allocated to Military Production. If instead the labor constraint binds strictly in the first period of war, all labor is allocated to the defense sector: $l_{m,0}^{S,C} = \bar{l}^S$. In that case

$$m_0^{S,C} = (K_t^S)^\alpha (\bar{l}^S)^{1-\alpha},$$

and therefore

$$\frac{\partial \mathcal{D}_t}{\partial K_t^S} = \frac{\chi\beta}{1-\beta}\alpha(K_t^S)^{\alpha-1}(\bar{l}^S)^{1-\alpha} > 0.$$

The Boundary between the Two Cases. Let $a \equiv [B(1-\alpha)]^{(1-\alpha)/\alpha}$ and $K^\dagger \equiv \bar{l}^S/[B(1-\alpha)]^{1/\alpha}$. Combining the two cases gives

$$m_0^{S,C}(K) = \min\left\{aK,\ K^\alpha(\bar{l}^S)^{1-\alpha}\right\}.$$

The labor constraint is slack for $K < K^\dagger$ and binds strictly for $K > K^\dagger$. At $K = K^\dagger$, it binds with a zero multiplier. Output is continuous there, but the left and right derivatives of deterrence with respect to capital are

$$\mathcal{D}^-_{K,t}(K^\dagger, M_t^S) = Ba, \qquad \mathcal{D}^+_{K,t}(K^\dagger, M_t^S) = \alpha Ba.$$

Thus the ordinary derivative does not exist at this point, although both one-sided marginal deterrent values are positive. The derivative with respect to military capital remains $B(2-\delta_M)$. This is the point at which the two branches in (36) coincide; Appendix A.1 treats it by retaining both deterrence constraints.

A.3 Proof of Proposition 2

Private-Sector Equilibrium Conditions. Consider the defense-industry firm in country S. Taking as given prices $(p_{m,t}^S, p_{n,t}^S, w_t^S)$ and a per-unit subsidy s_t^S on the capital stock K_t^S, it chooses $\{i_t^S, l_{m,t}^S\}_{t\geq 0}$ to maximize discounted profits subject to the technology (5) and the law of motion (3). Let $\hat{\lambda}_{K,t}^S$ denote the firm's multiplier on the capital accumulation constraint. The firm's first-order conditions are

$$[l_{m,t}^S]: \qquad w_t^S = p_{m,t}^S(1-\alpha)(K_t^S)^\alpha(l_{m,t}^S)^{-\alpha}, \tag{51}$$

$$[i_t^S]: \qquad p_{n,t}^S = \hat{\lambda}_{K,t}^S, \tag{52}$$

$$[K_{t+1}^S]: \qquad \hat{\lambda}_{K,t}^S = \frac{1}{1+r_t^S}\left(s_{t+1}^S + p_{m,t+1}^S\alpha(K_{t+1}^S)^{\alpha-1}(l_{m,t+1}^S)^{1-\alpha} + (1-\delta_K)\hat{\lambda}_{K,t+1}^S\right). \tag{53}$$

The price of the numeraire good is normalized to one, $p_{n,t}^S = 1$. With the linear technology (4), profit maximization by numeraire producers gives $w_t^S = p_{n,t}^S = 1$ whenever $l_{n,t}^S > 0$, and with linear utility and positive consumption the household's first-order conditions give $1/(1+r_t^S) = \beta$. Using (52), this implies $\hat{\lambda}_{K,t}^S = 1$ for all t. Substituting these restrictions into (53) yields the firm Euler equation (29).

Decentralization We look for a decentralization of the planner allocation using a supporting price sequence for military goods satisfying

$$p^S_{m,t} = \lambda^S_{m,t} \qquad \forall t, \tag{54}$$

where $\lambda^S_{m,t}$ is the planner multiplier on (2). With (54), the firm Euler equation (29) becomes

$$1 = \beta\Big((1-\delta_K) + \lambda^S_{m,t+1}\alpha(K^S_{t+1})^{\alpha-1}(l^S_{m,t+1})^{1-\alpha} + s^S_{t+1}\Big). \tag{55}$$

Wedge Relative to the Planner and Closing It. Comparing (55) with the planner condition (50) shows that the capital-stock subsidy

$$s^S_t = \sum_j \gamma^S_{j,t}\frac{\partial \mathcal{D}^j_t}{\partial K^S_t}, \qquad t \geq 1, \tag{56}$$

exactly closes the deterrence wedge. The firm's optimal accumulation of K^S_{t+1} therefore coincides with the planner's for every $t \geq 0$. Given the same initial conditions and with lump-sum taxes available to finance the subsidy, the remaining equilibrium conditions coincide by construction, and the resulting competitive equilibrium implements the planner allocation.

For short wars, both capital derivatives vanish, so $s^S_t = 0$. For long wars at differentiable allocations, complementary slackness reduces (56) to $s^S_t = \gamma^S_t\,\partial\mathcal{D}_t/\partial K^S_t$, as stated in the proposition. At the long-war kink, both branch multipliers enter, giving $s^S_t = Ba\,\gamma^S_{I,t} + \alpha Ba\,\gamma^S_{F,t}$. These multipliers are determined jointly with the planner allocation. For long wars, the subsidy remains positive whenever $\gamma^S_t > 0$, including at the kink.

A.4 Proof of Proposition 5

The Two-Technology Deterrence Constraint. Fix a possible challenge date t. With two technologies, military output is $m^S_t = \sum_{g=1}^2 (K^S_{g,t})^{\alpha_g}(l^S_{g,t})^{1-\alpha_g}$ and $M^S_{t+1} = (1-\delta_M)M^S_t + m^S_t$. By (22), deterrence is

$$\mathcal{D}_t(K^S_{1,t}, K^S_{2,t}, M^S_t) = \frac{\chi\beta^{T-1}}{1-\beta}\sum_{\tau=0}^{T-1}\left(M^{S,C}_\tau - \bar{M}^A\right).$$

If wars are short, $T = 1$, only $M^{S,C}_0 = M^S_t$ enters this sum. Since technology-specific capital affects military production only after the possible challenge date, $\partial\mathcal{D}_t/\partial K^S_{g,t} = 0$

for each g.

If wars are long, $T = 2$, then, with $B \equiv \chi\beta/(1-\beta)$ as in Appendix A.2,

$$\mathcal{D}_t = B\left(M_t^S + M_1^{S,C} - 2\bar{M}^A\right), \quad M_1^{S,C} = (1-\delta_M)M_t^S + \sum_{g=1}^{2}(K_{g,t}^S)^{\alpha_g}\left(l_{g,0}^{S,C}\right)^{1-\alpha_g}.$$

Therefore technology-g capital contributes to deterrence through its effect on first-period conflict production $m_{g,0}^{S,C} = (K_{g,t}^S)^{\alpha_g}(l_{g,0}^{S,C})^{1-\alpha_g}$. Since numeraire consumption is positive in the first period of war, labor in technology g equates the marginal value of its output to its marginal cost, $B(1-\alpha_g)(K_{g,t}^S)^{\alpha_g}(l_{g,0}^{S,C})^{-\alpha_g} = w_g(l_{g,0}^{S,C})$. With $w_g(l) = l^{1/\varepsilon_{\ell,g}}$, conflict labor is proportional to $(K_{g,t}^S)^{\alpha_g/(\alpha_g+1/\varepsilon_{\ell,g})}$, so conflict production has elasticity $\alpha_g(1+\varepsilon_g)$ with respect to $K_{g,t}^S$. Hence

$$\frac{\partial \mathcal{D}_t}{\partial K_{g,t}^S} = B\alpha_g(1+\varepsilon_g)\frac{m_{g,0}^{S,C}}{K_{g,t}^S} > 0$$

for each active technology.

Firm and Planner FOCs. Consider technology g. Under the supporting price $p_{m,t}^S = \lambda_{m,t}^S$, the firm's Euler equation for $K_{g,t+1}^S$ and the corresponding planner Euler equation are

$$\begin{aligned}
\text{firm:} \quad & 1 = \beta\Big((1-\delta_K) + \lambda_{m,t+1}^S \alpha_g (K_{g,t+1}^S)^{\alpha_g - 1}(l_{g,t+1}^S)^{1-\alpha_g} + s_{g,t+1}^S\Big), \\
\text{planner:} \quad & 1 = \beta\Big((1-\delta_K) + \lambda_{m,t+1}^S \alpha_g (K_{g,t+1}^S)^{\alpha_g - 1}(l_{g,t+1}^S)^{1-\alpha_g} + \gamma_{t+1}^S \frac{\partial \mathcal{D}_{t+1}}{\partial K_{g,t+1}^S}\Big).
\end{aligned}$$

The labor conditions coincide under the supporting price, since the firm pays the wage $w_g(l_{g,t}^S)$ and the planner faces the same marginal cost. The only wedge is the deterrence value of technology-g capital. Setting

$$s_{g,t}^S = \gamma_t^S \frac{\partial \mathcal{D}_t}{\partial K_{g,t}^S}, \qquad t \geq 1,$$

closes this wedge for each $g \in \{1,2\}$ and implements the planner allocation.

Short and Long Wars. The shape of the deterrence constraint gives the rest of the result. If wars are short, the derivative with respect to $K_{g,t}^S$ is zero, so $s_{g,t}^S = 0$. If wars are long and the deterrence constraint has a positive shadow value, $\gamma_t^S > 0$, the derivative is strictly positive for each active technology, so $s_{g,t}^S > 0$. This proves the proposition.

A.5 Proof of Proposition 6

Setup. Consider the steady state of the two-technology economy when wars are long, and suppress country superscripts and time subscripts. Technology g produces $m_g = K_g^{\alpha_g} l_g^{1-\alpha_g}$ with labor supplied at the marginal cost $w_g(l_g) = l_g^{1/\varepsilon_{\ell,g}}$; l_g^C and m_g^C denote its labor and output in the first period of a war. By Proposition 5, $s_g = \gamma\, \partial \mathcal{D}/\partial K_g$. Since the deterrence constraint has a positive shadow value, $\gamma > 0$, so the comparison of subsidies reduces to a comparison of deterrence derivatives.

Conflict versus Peace. Let $B \equiv \chi\beta/(1-\beta)$ be the marginal value of first-period conflict production in a long war, as in Appendix A.2, and let p_m be the peacetime price of military goods. The conflict and peacetime labor FOCs for technology g are

$$B(1-\alpha_g)K_g^{\alpha_g}(l_g^C)^{-\alpha_g} = (l_g^C)^{1/\varepsilon_{\ell,g}}, \qquad p_m(1-\alpha_g)K_g^{\alpha_g} l_g^{-\alpha_g} = l_g^{1/\varepsilon_{\ell,g}}.$$

Hence $l_g^C/l_g = (B/p_m)^{1/(\alpha_g + 1/\varepsilon_{\ell,g})}$ and $m_g^C/m_g = (l_g^C/l_g)^{1-\alpha_g} = (B/p_m)^{\varepsilon_g}$. The assumption that military production rises during conflict implies $B/p_m > 1$.

The Deterrence Derivative. In a long war, technology-g capital affects deterrence by changing first-period conflict production. By the conflict labor FOC, l_g^C is proportional to $K_g^{\alpha_g/(\alpha_g + 1/\varepsilon_{\ell,g})}$, so conflict production has elasticity $\alpha_g(1+\varepsilon_g)$ with respect to K_g and

$$\frac{\partial \mathcal{D}}{\partial K_g} = B\alpha_g(1+\varepsilon_g)\frac{m_g^C}{K_g}.$$

Using the peacetime capital Euler equation, $p_m \alpha_g m_g/K_g = \beta^{-1} - (1-\delta_K) - s_g$, and substituting for m_g^C/m_g gives

$$\frac{\partial \mathcal{D}}{\partial K_g} = \Phi(\varepsilon_g)\left(\beta^{-1} - (1-\delta_K) - s_g\right), \qquad \Phi(\varepsilon) \equiv (1+\varepsilon)\left(\frac{B}{p_m}\right)^{1+\varepsilon}.$$

Because $B/p_m > 1$, $\Phi(\varepsilon)$ is strictly increasing in ε.

Comparing Subsidies. Combining $s_g = \gamma\, \partial \mathcal{D}/\partial K_g$ with the previous display gives

$$s_g = \gamma\Phi(\varepsilon_g)\left(\beta^{-1} - (1-\delta_K) - s_g\right), \quad \text{so} \quad s_g = \frac{\gamma\Phi(\varepsilon_g)}{1+\gamma\Phi(\varepsilon_g)}\left(\beta^{-1} - (1-\delta_K)\right).$$

Since $\beta^{-1} - (1 - \delta_K) > 0$, the right-hand side is strictly increasing in $\Phi(\varepsilon_g)$, and Φ is strictly increasing in ε_g. Therefore $\varepsilon_1 > \varepsilon_2$ implies $s_1 > s_2$. Restoring time subscripts gives $s^S_{1,t} > s^S_{2,t}$.

B Extensions

B.1 Traditional Rational Deterrence Theory

Geopolitical States. Fix a possible challenge date t. The baseline model assumes that if Adversary challenges the status quo, Sovereign fights. This extension makes that response explicit. There are three possible geopolitical states at t:

$$z_t \in \{P, B, C\}.$$

The first is peace, denoted by P, in which Adversary does not challenge and the status quo allocation of the public good remains in place. The second is backing down, denoted by B, in which Adversary challenges and Sovereign does not contest the challenge. The letter B stands for backing down. The third is conflict, denoted by C, in which Adversary challenges and Sovereign fights.

The desired equilibrium outcome is still peace. To sustain it, two incentive constraints must hold. First, Adversary must prefer not to challenge, taking as given that a challenge would be met by conflict. This is the deterrence constraint already studied in the baseline model. Second, if Adversary were to challenge, Sovereign must prefer fighting to backing down. This is the new commitment constraint. Throughout this extension, Sovereign cannot commit to its own future decisions: neither to fighting after a challenge nor to its future peacetime allocation, which it chooses anew at every date.

Peace. Let K^S_t and M^S_t denote the peacetime defense-industrial and military stocks at the possible challenge date, and let $V^{S,P}_t$ denote the consumption value of the peacetime continuation. Since Sovereign cannot commit to its future peacetime allocation, the continuation is chosen by Sovereign at date t and later, so $V^{S,P}_t$ is a function of the stocks it inherits. If the state is P, Sovereign's continuation welfare is

$$W^{S,P}_t(K^S_t, M^S_t) = \frac{\bar{G}^S}{1-\beta} + V^{S,P}_t(K^S_t, M^S_t). \tag{57}$$

Backing Down. If the state is B, Sovereign keeps the peacetime economic allocation but loses a present value $\Delta_t^S \geq 0$ of the public good:

$$W_t^{S,B}(K_t^S, M_t^S) = W_t^{S,P}(K_t^S, M_t^S) - \Delta_t^S. \tag{58}$$

This captures the idea that backing down preserves peace but concedes part of the disputed public good.

Conflict. If the state is C, the off-equilibrium continuation is the same wartime continuation as in the baseline model. Sovereign starts the conflict with the peacetime stocks (K_t^S, M_t^S), solves the wartime problem (20), and obtains

$$\begin{aligned} W_t^{S,C}(K_t^S, M_t^S) = \frac{\bar{G}^S}{1-\beta} + V_t^{S,C}(K_t^S, M_t^S) \\ + \frac{\chi \beta^{T-1}}{1-\beta} \sum_{\tau=0}^{T-1} \left(M_\tau^{S,C}(K_t^S, M_t^S) - \bar{M}^A \right), \end{aligned} \tag{59}$$

where $V_t^{S,C}$ is the consumption value of the optimal wartime continuation. The term with the summation is the present value of the change in Sovereign's public good generated by the conflict technology.

Deterrence Constraint. The deterrence constraint is unchanged. Since the conjectured response to a challenge is conflict, Adversary compares peace with the conflict continuation. Define

$$\mathcal{D}_t(K_t^S, M_t^S) \equiv W_t^{A,P} - W_t^{A,C}(K_t^S, M_t^S). \tag{60}$$

Adversary is deterred when

$$\mathcal{D}_t(K_t^S, M_t^S) \geq \eta_t^A. \tag{61}$$

Commitment Constraint. The new object is Sovereign's commitment constraint. Let $\bar{V}_t^S$ denote any exogenous commitment value, such as resolve, reputation, domestic audience costs, or other political costs of backing down. Sovereign is willing to fight after a challenge when

$$W_t^{S,C}(K_t^S, M_t^S) + \bar{V}_t^S \geq W_t^{S,B}(K_t^S, M_t^S). \tag{62}$$

Equivalently, define

$$\mathcal{C}_t(K_t^S, M_t^S) \equiv W_t^{S,C}(K_t^S, M_t^S) - W_t^{S,P}(K_t^S, M_t^S) + \Delta_t^S + \bar{V}_t^S. \tag{63}$$

The commitment constraint is

$$\mathcal{C}_t(K_t^S, M_t^S) \geq 0. \tag{64}$$

The two constraints play different roles. The deterrence constraint is Adversary's no-challenge condition. The commitment constraint is Sovereign's no-back-down condition. A deterrence equilibrium with limited commitment requires both.

Planning Problem. Sovereign chooses a peacetime allocation intended to implement the P outcome. The allocation must be feasible, must deter Adversary from challenging, and must make Sovereign's threatened response credible. As in the baseline problem, conflict is off the equilibrium path. The planner nevertheless has to choose peacetime stocks that make the off-equilibrium threat credible, because Adversary's no-challenge condition is evaluated under the conjecture that a challenge would be met by conflict. The planner takes the continuation value $V_t^{S,P}$ in the commitment constraint as given: it is determined by Sovereign's future decisions, to which Sovereign cannot commit, and in equilibrium it is the value of the peacetime path that the planning problem generates from the date-t stocks. The primal planning problem is therefore

$$\begin{aligned} \max_{\{c_t^S, y_{n,t}^S, y_{m,t}^S, l_{n,t}^S, l_{m,t}^S, i_t^S, m_t^S, K_{t+1}^S, M_{t+1}^S\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \left(\bar{G}^S + c_t^S \right) \\ \text{s.t.} \quad & \text{technology and market-clearing constraints (2)–(11)}, \\ & \mathcal{D}_t(K_t^S, M_t^S) \geq \eta_t^A, \quad \forall t \geq 1, \\ & \mathcal{C}_t(K_t^S, M_t^S) \geq 0, \quad \forall t, \\ & \text{non-negativity constraints, for all } t. \end{aligned} \tag{65}$$

First-Order Conditions. Let $\gamma_t^S \geq 0$ and $\kappa_t^S \geq 0$ be the multipliers on the date-t deterrence and commitment constraints. The optimality condition for military capital is

$$\lambda_{m,t}^S = \beta \left((1 - \delta_M) \lambda_{m,t+1}^S + \gamma_{t+1}^S \frac{\partial \mathcal{D}_{t+1}}{\partial M_{t+1}^S} + \kappa_{t+1}^S \frac{\partial \mathcal{C}_{t+1}}{\partial M_{t+1}^S} \right). \tag{66}$$

The optimality condition for defense-industrial capital is

$$1 = \beta \left((1 - \delta_K) + \lambda_{m,t+1}^S \alpha \left(K_{t+1}^S \right)^{\alpha - 1} \left(l_{m,t+1}^S \right)^{1-\alpha} + \gamma_{t+1}^S \frac{\partial \mathcal{D}_{t+1}}{\partial K_{t+1}^S} + \kappa_{t+1}^S \frac{\partial \mathcal{C}_{t+1}}{\partial K_{t+1}^S} \right). \tag{67}$$

Relative to the baseline model, each stock now has two strategic values. It can deter Adversary by lowering Adversary's payoff from conflict, and it can strengthen commitment by raising Sovereign's payoff from fighting relative to backing down.

Optimal Subsidy. The private Euler equation for defense-industrial capital is unchanged because firms only price the peacetime use of capital: it is (29). Using the same supporting price as in the baseline, $p^S_{m,t+1} = \lambda^S_{m,t+1}$, comparison with the planner's Euler equation gives the implementing subsidy, for $t \geq 1$,

$$s^S_t = \gamma^S_t \frac{\partial \mathcal{D}_t}{\partial K^S_t} + \kappa^S_t \frac{\partial \mathcal{C}_t}{\partial K^S_t}. \tag{68}$$

The first term is the baseline deterrence externality: firms do not internalize that capital raises the cost of war for Adversary. The second term is the commitment externality: firms also do not internalize that capital can make Sovereign more willing to fight after a challenge.

Characterizing the Commitment Derivative. Let $p^{S,z}_{K,t}$ denote the full beginning-of-period market value of an additional unit of installed capital in continuation $z \in \{P, C\}$, including current rental income and the capital subsidy. Indexing each continuation by calendar time, its valuation equation is

$$p^{S,z}_{K,t} = p^{S,z}_{m,t} \alpha (K^{S,z}_t)^{\alpha-1} (l^{S,z}_{m,t})^{1-\alpha} + s^{S,z}_t + \beta(1-\delta_K) p^{S,z}_{K,t+1}, \tag{69}$$

where $s^{S,P}_t = s^S_t$ and subsidies are zero along the conflict continuation.

Since Δ^S_t and $\bar{V}^S_t$ are exogenous, the commitment derivative is $\partial \mathcal{C}_t / \partial K^S_t = \partial W^{S,C}_t / \partial K^S_t - \partial W^{S,P}_t / \partial K^S_t$. At the supporting procurement prices, the wartime envelope and the firm's asset valuation satisfy the same recursion and transversality condition. Thus $\partial W^{S,C}_t / \partial K^S_t = p^{S,C}_{K,t}$.

The peace continuation is conditional on the date-t geopolitical decision having been resolved peacefully, with strategic constraints imposed from $t+1$ onward. Its envelope condition is

$$\frac{\partial W^{S,P}_t}{\partial K^S_t} = p^S_{m,t} \alpha (K^S_t)^{\alpha-1} (l^S_{m,t})^{1-\alpha} + \beta(1-\delta_K) \left[\frac{\partial W^{S,P}_{t+1}}{\partial K^S_{t+1}} + \gamma^S_{t+1} \frac{\partial \mathcal{D}_{t+1}}{\partial K^S_{t+1}} + \kappa^S_{t+1} \frac{\partial \mathcal{C}_{t+1}}{\partial K^S_{t+1}} \right]. \tag{70}$$

The second term is the discounted continuation value of surviving capital. With interior investment, the planner's investment first-order condition sets β times the bracketed term

equal to one. The firm's investment condition likewise gives $1 = \beta p^{S,P}_{K,t+1}$. Therefore,

$$\frac{\partial W^{S,P}_t}{\partial K^S_t} = p^S_{m,t}\alpha(K^S_t)^{\alpha-1}(l^S_{m,t})^{1-\alpha} + 1 - \delta_K = p^{S,P}_{K,t} - s^S_t. \tag{71}$$

The current subsidy enters the private asset price but is a domestic transfer. Consequently,

$$\frac{\partial \mathcal{C}_t}{\partial K^S_t} = p^{S,C}_{K,t} - p^{S,P}_{K,t} + s^S_t. \tag{72}$$

Substituting into $s^S_t = \gamma^S_t \, \partial \mathcal{D}_t / \partial K^S_t + \kappa^S_t \, \partial \mathcal{C}_t / \partial K^S_t$ and collecting terms gives, for $t \geq 1$ and $\kappa^S_t < 1$,

$$s^S_t = \frac{1}{1-\kappa^S_t}\left[\gamma^S_t \frac{\partial \mathcal{D}_t}{\partial K^S_t} + \kappa^S_t \left(p^{S,C}_{K,t} - p^{S,P}_{K,t}\right)\right]. \tag{73}$$

The Value of Capital in War. This expression separates two reasons for subsidizing defense-industrial capital. The deterrence term reflects capital's effect on Adversary's incentive to challenge. The commitment-related price term captures the change in capital's market value from peace to conflict: it is positive when capital appreciates in conflict. This gives the same asset-price comparison that determines optimal industrial policy in Kooi (2025). The difference is that that model is essentially static and price of capital corresponded to the spot market price rather than the discounted present value. The factor $1/(1-\kappa^S_t)$ shows up because the subsidy affects the peacetime asset price. Along an interior peacetime investment path, $p^{S,P}_{K,t} = \beta^{-1}$ for $t \geq 1$.

B.2 Equilibrium Wars

We add shocks to Adversary's willingness to attack and allow Sovereign to choose the war probabilities it implements. As in the baseline, Sovereign commits at date 0, before the first attack shock, to its policy path conditional on peace surviving. If war starts, it solves the baseline wartime problem given the inherited stocks. War can occur only once.

B.2.1 War Probabilities and the Planning Problem

The economic environment and Adversary's stock-replacement rules are unchanged. Write $W^{S,C}_t(K^S_t, M^S_t)$ and $W^{A,C}_t(K^S_t, M^S_t)$ for the baseline conflict values. At each peacetime challenge, Adversary independently draws $\eta^A_t \sim F_t$ and receives this additional payoff if it attacks. Assume a continuous positive density f_t over the relevant range and uniformly bounded first moments.

Let $\mathbf{K}_t^S = \{K_u^S\}_{u \geq t}$ and $\mathbf{M}_t^S = \{M_u^S\}_{u \geq t}$ denote the peacetime stock paths. Conditional on not attacking at t, Adversary's value is the bounded solution to

$$W_t^{A,P} = \bar{G}^A + \bar{c}^A + \beta \int \max\left\{W_{t+1}^{A,P}(\mathbf{K}_{t+1}^S, \mathbf{M}_{t+1}^S), W_{t+1}^{A,C}(K_{t+1}^S, M_{t+1}^S) + \eta\right\} dF_{t+1}(\eta). \quad (74)$$

Thus peace today preserves the option to attack later. As in the baseline, deterrence is the difference between Adversary's peace and conflict values:

$$\mathcal{D}_t(\mathbf{K}_t^S, \mathbf{M}_t^S) = W_t^{A,P}(\mathbf{K}_t^S, \mathbf{M}_t^S) - W_t^{A,C}(K_t^S, M_t^S). \quad (75)$$

Adversary attacks when $\eta_t^A > \mathcal{D}_t$. Conditional on reaching date t without war, the probabilities of war and peace are

$$P_t^C = 1 - F_t(\mathcal{D}_t), \qquad P_t^P = F_t(\mathcal{D}_t). \quad (76)$$

The unconditional probability that peace survives through t is $\mathbb{P}_{0,t}^P$; the probability that war first starts at t is $\mathbb{P}_{0,t}^C$. They satisfy

$$\mathbb{P}_{0,t}^P = \mathbb{P}_{0,t-1}^P P_t^P, \qquad \mathbb{P}_{0,t}^C = \mathbb{P}_{0,t-1}^P P_t^C, \qquad \mathbb{P}_{0,-1}^P = 1. \quad (77)$$

Households and firms take these probabilities as given. Under the positive-consumption conditions used below, the date-0 state prices are

$$Q_t^P = \beta^t \mathbb{P}_{0,t}^P, \qquad Q_{t,\tau}^C = \beta^{t+\tau} \mathbb{P}_{0,t}^C, \quad (78)$$

where $C_{t,\tau}$ is the conflict continuation τ periods after war starts at t.

Sovereign chooses the peacetime allocation and the probabilities it implements:

$$\begin{aligned} \max_{\{c_t^S, l_{m,t}^S, i_t^S, m_t^S, K_{t+1}^S, M_{t+1}^S, P_t^C\}_{t \geq 0}} \quad & \sum_{t=0}^{\infty} \beta^t \left[\mathbb{P}_{0,t}^P(\bar{G}^S + c_t^S) + \mathbb{P}_{0,t}^C W_t^{S,C}(K_t^S, M_t^S)\right] \\ \text{s.t.} \quad & \mathcal{D}_t(\mathbf{K}_t^S, \mathbf{M}_t^S) = F_t^{-1}(1 - P_t^C), \qquad t \geq 0, \end{aligned} \quad (79)$$

together with the baseline resource, production, and accumulation constraints, non-negativity, and given initial stocks. The last constraint ensures that the chosen probabilities agree with Adversary's optimal decisions. We first derive the subsidy conditional on a feasible probability path, then use the additional probability first-order conditions to characterize its shadow value.

B.2.2 Optimal Industrial Policy

We use the existing economic multipliers, with present-value weights Q_t^P in peace, so the supporting price remains $p_{m,t}^S = \lambda_{m,t}^S$. We consider differentiable allocations with interior choices and positive numeraire consumption.

Attach $\beta^u \xi_u^S$ to the probability constraint $\mathcal{D}_u - F_u^{-1}(1 - P_u^C) = 0$ in (79). Differentiating these constraints contributes $\sum_{u=0}^{t} \beta^u \xi_u^S \, \partial \mathcal{D}_u / \partial K_t^S$ to the first-order condition for K_t^S. Holding other dates' stocks fixed,

$$\frac{\partial \mathcal{D}_t}{\partial K_t^S} = -\frac{\partial W_t^{A,C}}{\partial K_t^S} = \frac{\chi \beta^{T-1}}{1-\beta} \sum_{\tau=0}^{T-1} \frac{\partial M_{t,\tau}^{S,C}}{\partial K_t^S}. \tag{80}$$

For $u < t$, the envelope theorem applied to (74) gives

$$\frac{\partial \mathcal{D}_u}{\partial K_t^S} = -\beta^{t-u} \left(\prod_{v=u+1}^{t-1} P_v^P \right) P_t^C \frac{\partial \mathcal{D}_t}{\partial K_t^S}, \tag{81}$$

with an empty product equal to one. Thus the constraints' contribution is $Q_t^P \gamma_t^S \, \partial \mathcal{D}_t / \partial K_t^S$, where

$$\gamma_t^S \equiv \frac{1}{\mathbb{P}_{0,t}^P} \left[\xi_t^S - P_t^C \sum_{u=0}^{t-1} \xi_u^S \prod_{v=u+1}^{t-1} P_v^P \right]. \tag{82}$$

This groups the constraint multipliers into a net shadow value; it introduces no additional parameter. Dividing the capital first-order condition by Q_{t-1}^P gives, for $t \geq 1$,

$$1 = \beta P_t^P \left[1 - \delta_K + \lambda_{m,t}^S \alpha (K_t^S)^{\alpha-1} (l_{m,t}^S)^{1-\alpha} + \gamma_t^S \frac{\partial \mathcal{D}_t}{\partial K_t^S} \right] + \beta P_t^C \frac{\partial W_t^{S,C}}{\partial K_t^S}. \tag{83}$$

The firm's corresponding condition is

$$1 = \beta P_t^P \left[1 - \delta_K + p_{m,t}^S \alpha (K_t^S)^{\alpha-1} (l_{m,t}^S)^{1-\alpha} + s_t^S \right] + \beta P_t^C p_{K,t,0}^{S,C}. \tag{84}$$

Here $p_{K,t,0}^{S,C}$ is the full market value of installed capital when war starts. With zero wartime capital subsidies and procurement prices supporting the optimal wartime allocation, the firm's valuation and the planner's envelope give

$$p_{K,t,0}^{S,C} = \sum_{\tau=0}^{\infty} \left[\beta (1 - \delta_K) \right]^\tau p_{m,t,\tau}^{S,C} \alpha (K_{t,\tau}^{S,C})^{\alpha-1} (l_{m,t,\tau}^{S,C})^{1-\alpha} = \frac{\partial W_t^{S,C}}{\partial K_t^S}, \tag{85}$$

under the usual transversality conditions. Comparing (83) and (84) therefore yields

$$\boxed{s_t^S = \gamma_t^S \frac{\partial \mathcal{D}_t}{\partial K_t^S}.} \tag{86}$$

Why the Formula Is Unchanged. A positive probability of war makes firms value capital's productive use during war. Optimal wartime procurement prices convey that value, so it appears in both the private and planner conditions and cancels from the subsidy formula. Firms still take war probabilities as given and receive no payment for improving deterrence. Capital's marginal deterrent contribution is the same as in the baseline. Its shadow value now includes the effect of future strength on earlier attack decisions.

B.2.3 Characterizing the Shadow Value of Deterrence

The additional first-order conditions for war probabilities determine the multipliers ξ_t^S and hence γ_t^S. Let $W_t^{S,P}$ denote Sovereign's continuation value conditional on peace at t:

$$W_t^{S,P} = \bar{G}^S + c_t^S + \beta \left[P_{t+1}^P W_{t+1}^{S,P} + P_{t+1}^C W_{t+1}^{S,C} \right]. \tag{87}$$

Holding allocations and the other probabilities fixed, the first-order condition for P_t^C is

$$0 = \mathbb{P}_{0,t-1}^P (W_t^{S,C} - W_t^{S,P}) + \frac{\xi_t^S}{f_t(\mathcal{D}_t)}, \tag{88}$$

where $dF_t^{-1}(1 - P_t^C)/dP_t^C = -1/f_t(\mathcal{D}_t)$. Hence

$$\xi_t^S = \mathbb{P}_{0,t-1}^P f_t(\mathcal{D}_t) \left(W_t^{S,P} - W_t^{S,C} \right). \tag{89}$$

Substituting into the definition (82) gives

$$\gamma_t^S = \frac{f_t(\mathcal{D}_t)}{P_t^P} \left(W_t^{S,P} - W_t^{S,C} \right) - \frac{P_t^C}{P_t^P} \sum_{u=0}^{t-1} \frac{f_u(\mathcal{D}_u)}{P_u^P} \left(W_u^{S,P} - W_u^{S,C} \right). \tag{90}$$

The first term values the reduction in war risk at t. The second accounts for earlier attack decisions: stronger defenses at t lower Adversary's value of waiting and can encourage an earlier attack. Consequently, even when Sovereign prefers peace and capital deters at t, the net shadow value γ_t^S need not be positive.

B.3 Endogenous War Duration

This appendix extends the baseline model by allowing the duration of a conflict to depend on the military balance at the possible challenge date. The point is to show that the sufficient statistic for industrial policy is unchanged, but that the optimal subsidy is lower when Adversary assigns positive probability to a short war, under a natural condition.

Duration. Fix a possible challenge date t. In the baseline model, the duration of the off-equilibrium conflict continuation is exogenous. Here, the continuation lasts two periods with probability

$$\pi_t = \Pr(T = 2 \mid M_t^S) = \pi(M_t^S), \tag{91}$$

where π is differentiable, and we focus on the interior region in which $\pi_t \in (0,1)$. We assume the duration shock is realized at the possible challenge date, so the conditional conflict continuation is solved as in the baseline model once the duration is known.

Deterrence. Let $\mathcal{D}_t^1$ denote deterrence if the conflict continuation lasts one period and $\mathcal{D}_t^2$ denote deterrence if it lasts two periods. By (22),

$$\mathcal{D}_t^1 = \frac{\chi}{1-\beta}\left(M_t^S - \bar{M}^A\right), \qquad \mathcal{D}_t^2 = \frac{\chi\beta}{1-\beta}\left[M_t^S + M_1^{S,C} - 2\bar{M}^A\right].$$

The deterrence constraint is therefore

$$\mathcal{D}_t(K_t^S, M_t^S) = (1-\pi_t)\,\mathcal{D}_t^1 + \pi_t \mathcal{D}_t^2. \tag{92}$$

This nests the baseline model as the limiting cases $\pi_t = 0$ and $\pi_t = 1$.

Marginal Deterrent Values. We focus on the case in which some labor remains in numeraire production in the first period of the long-war continuation. Conditional on the continuation lasting two periods, the first-period military production problem is the same as in the baseline long-war case, so

$$m_0^{S,C}(K_t^S) = \left(\frac{\chi\beta}{1-\beta}(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}} K_t^S.$$

Using $M_1^{S,C} = (1-\delta_M)M_t^S + m_0^{S,C}$, the marginal deterrent values are

$$\frac{\partial \mathcal{D}_t}{\partial K_t^S} = \pi_t \left(\frac{\chi\beta}{1-\beta}\right)^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}, \tag{93}$$

$$\frac{\partial \mathcal{D}_t}{\partial M_t^S} = (1-\pi_t)\frac{\chi}{1-\beta} + \pi_t \frac{\chi\beta}{1-\beta}\left(1+(1-\delta_M)\right) + \pi'(M_t^S)\left(\mathcal{D}_t^2 - \mathcal{D}_t^1\right). \tag{94}$$

The industrial-capital derivative is the long-war derivative scaled by the probability that the second period is reached. Military capital has an additional duration effect because it changes this probability.

Optimal Subsidy. The sufficient statistic for industrial policy is unchanged. Proposition 2 applies to any differentiable deterrence constraint, so the optimal capital subsidy is still

$$s_t^S = \gamma_t^S \frac{\partial \mathcal{D}_t}{\partial K_t^S}.$$

Endogenous duration changes the derivative that enters this expression, not the expression itself.

Subsidies and the Private Steady State. To obtain a sharper comparative static, we characterize the steady state of the planning problem, in which the deterrence constraint binds and the derivatives of $\mathcal{D}$ are evaluated at the steady-state stocks. Steady-state values carry no time subscript or country superscript, $\rho \equiv \beta^{-1} - 1$ is the steady-state interest rate, and $\omega \equiv l_m/K$ is the labor-to-capital ratio in the defense sector. The two results of this and the next paragraph hold for any differentiable deterrence function with $\partial\mathcal{D}/\partial M^S > 0$, so they also apply to the extension of Appendix B.4. In a steady-state competitive equilibrium, the firm's labor and capital conditions are $1 = p_m(1-\alpha)\omega^{-\alpha}$ and $\rho + \delta_K - s = p_m \alpha\, \omega^{1-\alpha}$. Combining them writes the subsidy as a decreasing function of ω; as $\beta \to 1$ it reduces to $s = \delta_K - \alpha\omega/(1-\alpha)$, the case used in the main text:

$$s = \rho + \delta_K - \frac{\alpha}{1-\alpha}\,\omega. \tag{95}$$

The Efficient Capital Mix. It remains to characterize how the deterrence function affects ω. At constant stocks, the planner's conditions of Appendix A.1 for defense-sector labor, military capital, and defense capital are $1 = \lambda_m(1-\alpha)\omega^{-\alpha}$, $(\rho+\delta_M)\lambda_m = \gamma\,\partial\mathcal{D}/\partial M^S$, and $\rho + \delta_K = \lambda_m \alpha\, \omega^{1-\alpha} + \gamma\,\partial\mathcal{D}/\partial K^S$. As $\beta \to 1$ these are the conditions for choosing K and ω to minimize the resource cost $(\delta_K + \omega)K$ subject to deterrence, the problem studied

in the main text. Eliminating λ_m and γ gives a relation whose left-hand side is increasing in both ω and the ratio of the marginal deterrent values, $(\partial\mathcal{D}/\partial K^S)/(\partial\mathcal{D}/\partial M^S)$. Therefore ω is decreasing in this ratio, and by (95) the optimal subsidy is increasing in it:

$$(\rho+\delta_M)\,\frac{\partial\mathcal{D}/\partial K^S}{\partial\mathcal{D}/\partial M^S}\,\omega^{\alpha}+\alpha\,\omega=(1-\alpha)(\rho+\delta_K). \tag{96}$$

Comparative Static. Let the superscript L denote the long-war-for-sure case and E the endogenous-duration case. From (93),

$$\left(\frac{\partial\mathcal{D}}{\partial K^S}\right)^E=\pi\left(\frac{\partial\mathcal{D}}{\partial K^S}\right)^L.$$

For military capital, (94) implies

$$\left(\frac{\partial\mathcal{D}}{\partial M^S}\right)^E=\pi\left(\frac{\partial\mathcal{D}}{\partial M^S}\right)^L+(1-\pi)\frac{\chi}{1-\beta}+\pi'(M)\left(\mathcal{D}^2-\mathcal{D}^1\right).$$

Thus

$$\frac{\left(\partial\mathcal{D}/\partial K^S\right)^E}{\left(\partial\mathcal{D}/\partial M^S\right)^E}<\frac{\left(\partial\mathcal{D}/\partial K^S\right)^L}{\left(\partial\mathcal{D}/\partial M^S\right)^L}$$

whenever

$$(1-\pi)\frac{\chi}{1-\beta}+\pi'(M)\left(\mathcal{D}^2-\mathcal{D}^1\right)>0.$$

This condition is automatically satisfied whenever $\pi'(M)$ and $\mathcal{D}^2-\mathcal{D}^1$ have the same sign or one of them is zero; in particular, when a larger military stock makes a long war more likely and long wars are at least as good for deterrence as short wars, $\mathcal{D}^2\geq\mathcal{D}^1$. In that case, endogenous duration lowers the ratio of marginal deterrent values. Equation (96) then implies that ω is higher than in the long-war-for-sure case. Finally, (95) implies that the optimal capital subsidy is lower.

B.4 Strategic Interactions

This appendix extends the baseline model by allowing for a limited strategic interaction in which Adversary responds to Sovereign's choices of K^S_{t+1} and M^S_{t+1}. We focus on a simple interaction in which Adversary chooses M^A_{t+1} according to a linear rule. This allows us to study Sovereign's optimal policy when Adversary responds strategically, while staying

close to the baseline model in which there is no war on the equilibrium path.[22]

B.4.1 Setup

Response Rule. At date t, Sovereign chooses the stocks that will be available at the next possible challenge date, K^S_{t+1} and M^S_{t+1}. After observing these stocks, Adversary sets its next-period military stock according to

$$M^A_{t+1} = \bar{M}^A + \iota_K K^S_{t+1} + \iota_M M^S_{t+1}, \tag{97}$$

where $\iota_K, \iota_M \geq 0$. The coefficient ι_K measures how much Adversary expands its military stock in response to Sovereign's defense-industrial capital, while ι_M measures the response to Sovereign's military capital. We assume the parameters are such that it is feasible for Adversary to follow this rule, and that the equipment Adversary adds in response is financed from an exogenous endowment rather than from its consumption, so that Adversary's consumption remains constant, as in the baseline model. Without this assumption, Adversary's consumption would differ between the peace continuation, in which it keeps adjusting its stock, and the conflict continuation, in which its stock is fixed, and the deterrence function would include the resource cost of the response in addition to the military balance.

Wars last either one or two periods, $T \in \{1, 2\}$. During a conflict that starts at t, Adversary's military stock is fixed at its stock at the possible challenge date:

$$M^{A,C}_\tau = M^A_t \qquad \text{for all } \tau \in \{0, 1\} \text{ reached during the conflict.}$$

Response-Adjusted Deterrence. Let $\mathcal{D}_t(K^S_t, M^S_t; M^A_t)$ denote the baseline deterrence function when Adversary's military stock at the possible challenge date is treated as a parameter. With the response rule (97), the deterrence function is

$$\widetilde{\mathcal{D}}_t(K^S_t, M^S_t) \equiv \mathcal{D}_t\left(K^S_t, M^S_t; \bar{M}^A + \iota_K K^S_t + \iota_M M^S_t\right). \tag{98}$$

The deterrence constraint is therefore

$$\widetilde{\mathcal{D}}_t(K^S_t, M^S_t) \geq \eta^A_t. \tag{99}$$

[22] Fully endogenizing Adversary's decision would require changing the model in several ways because, in a deterrence equilibrium, Adversary never uses its military stockpile in war and hence has no direct reason to accumulate it. One would therefore have to introduce other motives for holding a stockpile of military equipment, such as a positive probability of war or the possibility that Sovereign or third parties attack Adversary.

The distinction from the baseline is that Sovereign now internalizes how its peacetime choices affect Adversary's military stock, and hence the tightness of the deterrence constraint.

Marginal Deterrent Values. The response rule changes the marginal deterrent values through a chain-rule term. Suppressing time subscripts and evaluating derivatives at the relevant possible challenge date,

$$\frac{\partial \widetilde{\mathcal{D}}}{\partial K^S} = \frac{\partial \mathcal{D}}{\partial K^S} + \iota_K \frac{\partial \mathcal{D}}{\partial M^A}, \tag{100}$$

$$\frac{\partial \widetilde{\mathcal{D}}}{\partial M^S} = \frac{\partial \mathcal{D}}{\partial M^S} + \iota_M \frac{\partial \mathcal{D}}{\partial M^A}. \tag{101}$$

Since a higher adversary military stock makes conflict more attractive to Adversary, $\partial \mathcal{D}/\partial M^A < 0$. Hence an increase in ι_K lowers the deterrent value of Sovereign's defense-industrial capital, and an increase in ι_M lowers the deterrent value of Sovereign's military capital.

B.4.2 Optimal Policy

Planning Problem. Sovereign chooses the peacetime allocation taking the response rule as given. Since the response rule is already embedded in $\widetilde{\mathcal{D}}_t$, Adversary's military stock does not need to be tracked as an additional constraint in the reduced-form primal problem. The planning problem is

$$\begin{aligned} \max_{\{c_t^S, y_{n,t}^S, y_{m,t}^S, l_{n,t}^S, l_{m,t}^S, i_t^S, m_t^S, K_{t+1}^S, M_{t+1}^S\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \left(\bar{G}^S + c_t^S \right) \\ \text{s.t.} \quad & \text{technology and market-clearing constraints (2)–(11),} \\ & \widetilde{\mathcal{D}}_t(K_t^S, M_t^S) \geq \eta_t^A, \quad \forall t \geq 1, \\ & \text{non-negativity constraints, for all } t. \end{aligned} \tag{102}$$

First-Order Conditions. Let $\gamma_t^S \geq 0$ be the multiplier on the date-t deterrence constraint. The optimality conditions for military capital and defense-industrial capital are

$$\lambda_{m,t}^S = \beta \left((1-\delta_M)\lambda_{m,t+1}^S + \gamma_{t+1}^S \frac{\partial \widetilde{\mathcal{D}}_{t+1}}{\partial M_{t+1}^S} \right), \tag{103}$$

$$1 = \beta \left((1-\delta_K) + \lambda_{m,t+1}^S \alpha \left(K_{t+1}^S\right)^{\alpha-1} \left(l_{m,t+1}^S\right)^{1-\alpha} + \gamma_{t+1}^S \frac{\partial \widetilde{\mathcal{D}}_{t+1}}{\partial K_{t+1}^S} \right). \tag{104}$$

The only difference from the baseline FOCs is that the planner uses the total derivatives in (100) and (101), rather than the partial derivatives holding Adversary's military stock fixed.

Optimal Subsidy. The private Euler equation for defense-industrial capital, (29), is unchanged. Comparing it with (104), and using the same supporting price for military goods as in the baseline, gives the implementing subsidy, for $t \geq 1$,

$$s_t^S = \gamma_t^S \frac{\partial \widetilde{\mathcal{D}}_t}{\partial K_t^S} = \gamma_t^S \left(\frac{\partial \mathcal{D}_t}{\partial K_t^S} + \iota_K \frac{\partial \mathcal{D}_t}{\partial M_t^A} \right). \tag{105}$$

Thus the sufficient statistic is unchanged in form: the subsidy equals the shadow value of deterrence times the marginal contribution of capital to deterrence. The response rule changes the relevant marginal contribution. Since $\partial \mathcal{D}/\partial M^A < 0$, a stronger adversary response to K^S directly lowers the marginal deterrent value of defense-industrial capital.

B.4.3 Comparative Static: Result

Statement. We now ask how the optimal steady-state capital subsidy changes with the reaction coefficients ι_K and ι_M. Focus on long wars ($T = 2$) and an interior steady-state region in which the labor constraint is slack in the first period of war and the response-adjusted marginal deterrent values of both K^S and M^S are positive. Let s denote the steady-state capital subsidy. In this region, increasing the response to Sovereign's defense-industrial capital lowers the optimal capital subsidy, while increasing the response to Sovereign's military capital raises it:

$$\frac{\partial s}{\partial \iota_K} < 0, \qquad \frac{\partial s}{\partial \iota_M} > 0. \tag{106}$$

The intuition is simple. A higher ι_K means Adversary arms more in response to Sovereign's defense-industrial capital. This makes defense-industrial capital less effective

at providing deterrence, so the planner tilts away from it and the capital subsidy falls. A higher ι_M means Adversary arms more in response to Sovereign's military capital. This makes military capital less effective at providing deterrence relative to the industrial base, so the planner tilts toward defense-industrial capital and the capital subsidy rises.

B.4.4 Comparative Static: Derivation

This section derives the comparative static in (106). Throughout we focus on the case where wars are long.

Deterrent Values. In a long war, the baseline deterrence function (22), with Adversary's stock M_t^A in place of $\bar{M}^A$, is

$$\mathcal{D}_t(K_t^S, M_t^S; M_t^A) = \frac{\chi\beta}{1-\beta}\left[M_t^S + M_1^{S,C} - 2M_t^A\right].$$

Thus

$$\frac{\partial \mathcal{D}}{\partial M^A} = -\frac{2\chi\beta}{1-\beta}.$$

Equations (100) and (101) become

$$\frac{\partial \widetilde{\mathcal{D}}}{\partial K^S} = \frac{\partial \mathcal{D}}{\partial K^S} - \frac{2\chi\beta}{1-\beta}\iota_K, \tag{107}$$

$$\frac{\partial \widetilde{\mathcal{D}}}{\partial M^S} = \frac{\partial \mathcal{D}}{\partial M^S} - \frac{2\chi\beta}{1-\beta}\iota_M. \tag{108}$$

Signing the Ratio. The steady-state conditions of Appendix B.3 apply with $\widetilde{\mathcal{D}}$ in place of $\mathcal{D}$, so the optimal subsidy is increasing in the response-adjusted ratio of marginal deterrent values,

$$\frac{\partial \widetilde{\mathcal{D}}/\partial K^S}{\partial \widetilde{\mathcal{D}}/\partial M^S} = \frac{\partial \mathcal{D}/\partial K^S + \iota_K \partial \mathcal{D}/\partial M^A}{\partial \mathcal{D}/\partial M^S + \iota_M \partial \mathcal{D}/\partial M^A}.$$

Because $\partial \mathcal{D}/\partial M^A < 0$, and because we focus on the interior region in which the numerator and denominator are positive,

$$\frac{\partial}{\partial \iota_K}\left(\frac{\partial \widetilde{\mathcal{D}}/\partial K^S}{\partial \widetilde{\mathcal{D}}/\partial M^S}\right) = \frac{\partial \mathcal{D}/\partial M^A}{\partial \widetilde{\mathcal{D}}/\partial M^S} < 0,$$

$$\frac{\partial}{\partial \iota_M}\left(\frac{\partial \widetilde{\mathcal{D}}/\partial K^S}{\partial \widetilde{\mathcal{D}}/\partial M^S}\right) = -\frac{\left(\partial \widetilde{\mathcal{D}}/\partial K^S\right)\left(\partial \mathcal{D}/\partial M^A\right)}{\left(\partial \widetilde{\mathcal{D}}/\partial M^S\right)^2} > 0.$$

Equation (96) then implies that increasing ι_K raises ω, while increasing ι_M lowers ω. Finally, (95) implies that increasing ι_K lowers the optimal capital subsidy, while increasing ι_M raises it.

C Quantitative Model

C.1 Competitive Equilibrium

In this section, we present the optimization problem of households, producers, and the cost-minimization problem of the government. We show their optimality conditions, and we define the competitive equilibrium used in the model presented in Section 4.

Household's Problem The household holds a risk-free bond B_t in zero net supply and takes as given the goods prices $p_{g,t}$, the sectoral wages $w_{g,t}$, the interest rate r_t, the dividends Π_t of the producers it owns, and the lump-sum tax T_t. The price index of the consumption basket is the numeraire. With C_t and $\tilde{L}_t$ the aggregators of Section 4, $B_0 = 0$, and a no-Ponzi condition, it solves:

$$\max_{\{c_{g,t}, L_{g,t}, B_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big(\log C_t - \frac{\tilde{L}_t^{1+1/\xi}}{1+1/\xi} + G_t \Big)$$
$$\text{s.t.} \quad \sum_{g \in \mathcal{G}} p_{g,t} c_{g,t} + \frac{B_{t+1}}{1+r_t} = \sum_{g \in \mathcal{G}} w_{g,t} L_{g,t} + \Pi_t - T_t + B_t.$$

The demands for goods and the first-order conditions for the bond and for sectoral labor supply are:

$$c_{g,t} = \epsilon_g \Big(\frac{p_{g,t}}{P_{C,t}} \Big)^{-\sigma_C} C_t, \qquad P_{C,t} = \Big[\sum_{g \in \mathcal{G}} \epsilon_g \, p_{g,t}^{1-\sigma_C} \Big]^{\frac{1}{1-\sigma_C}} = 1, \tag{109}$$

$$\frac{1}{1+r_t} = \beta \frac{C_t}{C_{t+1}}, \qquad \frac{w_{g,t}}{C_t} = \tilde{L}_t^{1/\xi} \Big(\frac{L_{g,t}}{\tilde{L}_t} \Big)^{\frac{1}{\tau}}. \tag{110}$$

Producers' Problem The producer of sector g owns its capital stock and pays its profit as a dividend to the household. It takes as given the goods prices, the wage, the interest rate, and the subsidy rate $s_{g,t}$ on the replacement value $P_{g,t}^I K_{g,t}$ of its capital, where $P_{g,t}^I$ is the price of its investment basket. With $K_{g,0}$ given and $R_t = \prod_{v=0}^{t-1}(1+r_v)$ the cumulative

gross return, it solves:

$$\max_{\{L_{g,t},x_{g,r,t},i_{g,r,t},I_{g,t},K_{g,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \frac{1}{R_t}\Big(p_{g,t}Y_{g,t} + s_{g,t}P^I_{g,t}K_{g,t} - w_{g,t}L_{g,t} - \sum_{r\in\mathcal{G}} p_{r,t}\big(x_{g,r,t} + i_{g,r,t}\big)\Big)$$

$$\text{s.t.} \quad Y_{g,t} = \Big[\alpha_{V,g}\, V_{g,t}^{\frac{\sigma_Y-1}{\sigma_Y}} + \alpha_{X,g}\, X_{g,t}^{\frac{\sigma_Y-1}{\sigma_Y}}\Big]^{\frac{\sigma_Y}{\sigma_Y-1}},$$

$$V_{g,t} = \Big[\alpha_{K,g}\, K_{g,t}^{\frac{\sigma_{VA}-1}{\sigma_{VA}}} + \alpha_{L,g}\, L_{g,t}^{\frac{\sigma_{VA}-1}{\sigma_{VA}}}\Big]^{\frac{\sigma_{VA}}{\sigma_{VA}-1}}, \qquad X_{g,t} = \Big[\sum_{r\in\mathcal{G}} \alpha_{g,r}\, x_{g,r,t}^{\frac{\sigma_X-1}{\sigma_X}}\Big]^{\frac{\sigma_X}{\sigma_X-1}},$$

$$K_{g,t+1} = (1-\delta_g)K_{g,t} + I_{g,t} - \frac{\varphi}{2}\Big(\frac{I_{g,t}}{K_{g,t}} - \delta_g\Big)^2 K_{g,t}, \qquad I_{g,t} = \prod_{r\in\mathcal{G}} i_{g,r,t}^{\vartheta_{g,r}}, \qquad I_{g,t} \geq 0.$$

Labor and the intermediate composite are paid their marginal revenue products, and the demands for individual intermediate inputs and investment goods follow from cost minimization:

$$w_{g,t} = p_{g,t}\,\alpha_{V,g}\,\alpha_{L,g}\Big(\frac{Y_{g,t}}{V_{g,t}}\Big)^{\frac{1}{\sigma_Y}}\Big(\frac{V_{g,t}}{L_{g,t}}\Big)^{\frac{1}{\sigma_{VA}}}, \qquad P^X_{g,t} = p_{g,t}\,\alpha_{X,g}\Big(\frac{Y_{g,t}}{X_{g,t}}\Big)^{\frac{1}{\sigma_Y}}, \tag{111}$$

$$x_{g,r,t} = \alpha_{g,r}^{\sigma_X}\Big(\frac{p_{r,t}}{P^X_{g,t}}\Big)^{-\sigma_X} X_{g,t}, \qquad P^X_{g,t} = \Big[\sum_{r\in\mathcal{G}} \alpha_{g,r}^{\sigma_X}\, p_{r,t}^{1-\sigma_X}\Big]^{\frac{1}{1-\sigma_X}}, \tag{112}$$

$$i_{g,r,t} = \vartheta_{g,r}\,\frac{P^I_{g,t}I_{g,t}}{p_{r,t}}, \qquad P^I_{g,t} = \prod_{r\in\mathcal{G}}\Big(\frac{p_{r,t}}{\vartheta_{g,r}}\Big)^{\vartheta_{g,r}}. \tag{113}$$

The producer values its capital at its marginal revenue product plus the subsidy. With $q_{g,t}$ the shadow value of installed capital, the first-order condition for investment, the Euler equation for capital, and the law of motion of the capital stock are

$$\frac{P^I_{g,t}}{1-\varphi\big(I_{g,t}/K_{g,t} - \delta_g\big)} \;\geq\; q_{g,t}, \qquad I_{g,t} \geq 0, \qquad \text{with complementary slackness,} \tag{114}$$

$$\begin{aligned} q_{g,t} = \beta\,\frac{C_t}{C_{t+1}}\Big[&(1-\delta_g)\,q_{g,t+1} + s_{g,t+1}P^I_{g,t+1} + p_{g,t+1}\,\mathrm{MPK}_{g,t+1} \\ &+ q_{g,t+1}\,\frac{\varphi}{2}\Big(\Big(\frac{I_{g,t+1}}{K_{g,t+1}}\Big)^2 - \delta_g^2\Big)\Big], \end{aligned} \tag{115}$$

$$\mathrm{MPK}_{g,t} = \alpha_{V,g}\,\alpha_{K,g}\Big(\frac{Y_{g,t}}{V_{g,t}}\Big)^{\frac{1}{\sigma_Y}}\Big(\frac{V_{g,t}}{K_{g,t}}\Big)^{\frac{1}{\sigma_{VA}}},$$

$$K_{g,t+1} = (1-\delta_g)K_{g,t} + I_{g,t} - \frac{\varphi}{2}\Big(\frac{I_{g,t}}{K_{g,t}} - \delta_g\Big)^2 K_{g,t}. \tag{116}$$

Government The government buys each basket $m_{j,t}$ at minimum cost, at price $p_{m,j,t}$. It pays the subsidies, and balances its budget with the lump-sum tax; in the postwar phase the spending rule of Section 4 pins down $m_{j,t}$, and the government sets $s_{g,t} = 0$. Given the goods prices and the quantity $m_{j,t}$ of basket j, it solves:

$$\min_{\{m_{j,g,t}\}_{g\in\mathcal{G}_j}} \sum_{g\in\mathcal{G}_j} p_{g,t}\, m_{j,g,t} \qquad \text{s.t.} \quad \prod_{g\in\mathcal{G}_j} m_{j,g,t}^{\psi_{j,g}} = m_{j,t}.$$

The price index $p_{m,j,t}$ of a unit of basket j and the cost-minimizing demands are:

$$p_{m,j,t} = \prod_{g\in\mathcal{G}_j} \left(\frac{p_{g,t}}{\psi_{j,g}}\right)^{\psi_{j,g}}, \qquad m_{j,g,t} = \psi_{j,g}\, \frac{p_{m,j,t} m_{j,t}}{p_{g,t}}. \tag{117}$$

The government budget constraint and the military stocks' law of motion are:

$$T_t = \sum_{j\in\mathcal{J}} p_{m,j,t} m_{j,t} + \sum_{g\in\mathcal{G}} s_{g,t} P^I_{g,t} K_{g,t}, \tag{118}$$

$$M_{j,t+1} = (1-\delta_{j,t}) M_{j,t} + m_{j,t}. \tag{119}$$

Market Clearing Sectoral labor supply is equal to labor demand. Sectoral output is equal to consumption, intermediate, investment, and military demand:

$$Y_{g,t} = c_{g,t} + \sum_{b\in\mathcal{G}} x_{b,g,t} + \sum_{b\in\mathcal{G}} i_{b,g,t} + \sum_{j\in\mathcal{J}} m_{j,g,t}, \qquad g \in \mathcal{G}. \tag{120}$$

Definition 3 (Competitive equilibrium of the quantitative model)**.** *Given the government's purchases and subsidy rates* $\{m_{j,t}, s_{g,t}\}_{t=0}^{\infty}$ *and the initial stocks* $(\mathbf{K}_0, \mathbf{M}_0)$*, a competitive equilibrium is a sequence of quantities* $\{c_{g,t}, L_{g,t}, x_{g,r,t}, i_{g,r,t}, I_{g,t}, K_{g,t+1}, m_{j,g,t}, M_{j,t+1}\}_{t=0}^{\infty}$*, prices* $\{p_{g,t}, w_{g,t}, r_t, P^X_{g,t}, P^I_{g,t}, p_{m,j,t}\}_{t=0}^{\infty}$*, and taxes and dividends* $\{T_t, \Pi_t\}_{t=0}^{\infty}$ *that satisfies conditions* (109)–(120) *in every period.*

C.2 Wartime Equilibrium and the Deterrence Function

This section presents additional details of the deterrence block presented in Section 4. We first discuss how Adversary's economic cost of war is defined in the model. Next, we present Sovereign's wartime problem and its solution. Finally, we derive the simplified deterrence constraint.

Adversary's Economic Cost of War In peace Adversary's consumption and labor supply are constant at $\{C^A, \tilde{L}^A\}$; from the outbreak of a war of length T they follow the paths $\{C_s^{A,(T)}, \tilde{L}_s^{A,(T)}\}_{s\geq 0}$. Let U^A denote Adversary's peacetime period utility from consumption and labor, and $U_s^{A,(T)}$ its value along the war paths. We define them as:

$$U^A \equiv \log C^A - \frac{\left(\tilde{L}^A\right)^{1+1/\xi}}{(1+1/\xi)}$$
$$U_s^{A,(T)} \equiv \log C_s^{A,(T)} - \frac{\left(\tilde{L}_s^{A,(T)}\right)^{1+1/\xi}}{(1+1/\xi)}$$

Hence, the economic cost of the war is:

$$\mathcal{E}^{(T)} = \sum_{s=0}^{\infty} \beta^s \left(U^A - U_s^{A,(T)}\right).$$

Sovereign's Wartime Problem Sovereign's public good rises by the settlement from period T on. Let V^{post} denote the household's welfare in the postwar competitive equilibrium under the spending rule specified in Section 4, evaluated with the public good at its initial level G_0; the rise in the public good enters the objective through the settlement term. The wartime government takes the postwar purchases as given: they are an exogenous demand for sectoral goods, equal in equilibrium to the amounts the spending rule implies. Subject to the technology, market clearing, the laws of motion with the wartime depreciation rates δ_j^w, $m_{j,s} \geq 0$ and $I_{g,s} \geq 0$, the wartime government maximizes

$$\sum_{s=0}^{T-1} \beta^s \left(\log C_s - \frac{\tilde{L}_s^{1+1/\xi}}{1+1/\xi} + G_0\right) + \frac{\beta^T}{1-\beta} \sum_{s=0}^{T-1} \chi_s^{(T)} \left(M_s - M_s^A\right) + \beta^T V^{\text{post}}\left(\mathbf{K}_T, \mathbf{M}_T\right).$$

The wartime problem is a simple public good provision problem, where the government procures military stocks $m_{j,s}$ from markets to reach the first-best allocation. Hence, conditions (109)–(120) hold with $s_{g,s} = 0$, with the wartime rates δ_j^w in the laws of motion of the military stocks.

How does the government procure military stocks $m_{j,s}$ optimally? Let $\mu_{j,s}$ denote the value, in utility terms, of a unit of basket j procured in period s, which becomes available in period $s+1$. It is worth its contribution to the settlement in the period it arrives, through the marginal product $\phi_j M_{s+1}/M_{j,s+1}$ of the stock in military power, plus the value of the depreciated unit. The government procures $m_{j,s}$ such that its marginal benefit, $\mu_{j,s}$, is equal to its marginal cost, $\frac{p_{m,j,s}}{C_s}$. If the marginal cost is higher than the marginal

benefit at zero purchases, it will optimally set $m_{j,s} = 0$. The terminal condition is $\mu_{j,T-1} = 0$: purchases in the last period of the war arrive after the settlement and have no value. The first-order condition for procurement, for $s = 0, \ldots, T-2$, and the recursion for the shadow values are

$$\frac{p_{m,j,s}}{C_s} \geq \mu_{j,s}, \qquad m_{j,s} \geq 0, \qquad \text{with complementary slackness,} \quad s = 0, \ldots, T-2, \tag{121}$$

$$\mu_{j,s} = \frac{\beta^{T-s}}{1-\beta} \chi_{s+1}^{(T)} \phi_j \frac{M_{s+1}}{M_{j,s+1}} + \beta \left(1 - \delta_j^w\right) \mu_{j,s+1}, \qquad s = 0, \ldots, T-2, \qquad \mu_{j,T-1} = 0. \tag{122}$$

These conditions, together with the initial conditions $(\mathbf{K}_t, \mathbf{M}_t)$, pin down Sovereign's military power in period s of the war: $M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t)$. The purchases of the last period of the war arrive after the settlement, so the procurement condition would set them to zero; we let them follow the postwar rule instead, $p_{m,j,T-1} m_{j,T-1} = \theta_j \theta \, \text{GDP}_{T-1}$, so that procurement does not drop to zero before the postwar phase.

Deriving the Simplified Deterrence Constraint Adversary's welfare differs between peace and war through the change in the public good and through the economic cost of its mobilization. In peace the public good stays at its initial level; in a war of length T it is reduced from period T on by the settlement $\sum_{s=0}^{T-1} \chi_s^{(T)} (M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t) - M_s^A)$, the territory Sovereign gains; here $M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t)$ is Sovereign's military power in period s of the war. Adversary evaluates its welfare in war before its length is known, $W_t^{A,C} = \sum_T \tilde{\pi}_T(M_t) W_t^{A,C,(T)}$. Discounting the settlement from its first period, Adversary's welfare loss from a war of length T is

$$W_t^{A,P} - W_t^{A,C,(T)} = \frac{\beta^T}{1-\beta} \sum_{s=0}^{T-1} \chi_s^{(T)} \left(M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t) - M_s^A \right) + \mathcal{E}^{(T)}.$$

Hence, the simplified deterrence function becomes:

$$\begin{aligned} \mathcal{D}_t(\mathbf{K}_t, \mathbf{M}_t) &= W_t^{A,P} - \sum_T \tilde{\pi}_T(M_t) W_t^{A,C,(T)} \\ &= \sum_{T=1}^{\bar{T}} \tilde{\pi}_T(M_t) \left[\frac{\beta^T}{1-\beta} \sum_{s=0}^{T-1} \chi_s^{(T)} \left(M_s^{(T)}(\mathbf{K}_t, \mathbf{M}_t) - M_s^A \right) + \mathcal{E}^{(T)} \right], \end{aligned}$$

from which the simplified deterrence constraint in Section 4 follows immediately.

C.3 Calibration

In this section, we discuss additional details related to the calibration of the quantitative model in Section 4. We first discuss the calibration of the model's economic block, after which we discuss the calibration of the deterrence block.

C.3.1 Economic Block

Sectoral Composition The sectoral data are from the 2017 BEA input-output accounts at the detail level (U.S. Bureau of Economic Analysis, 2023b). Following Iovino et al. (2025), we drop finance and insurance, owner-occupied housing, and government industries, which leaves 378 private industries. We further drop eight retail trade industries that have no commodity sales in the use table. The remaining $N = 370$ industries are the sectors of the model.

Technology and Preference Weights The consumption weights ϵ_g are set so that the expenditure shares $p_g c_g / C$ in the postwar steady state equal the sector's share of personal consumption expenditures, non-defense government purchases, and exports in the 2017 BEA use table. The CES weights of the production nests, $\alpha_{V,g}$, $\alpha_{X,g}$, $\alpha_{K,g}$, $\alpha_{L,g}$, $\alpha_{g,r}$, are calibrated so that the cost shares in the postwar steady state match those in the data: the intermediate spending shares of the use table, the value-added share of gross output, and the labor share of value added. As a normalization, we set $\alpha_{V,g} + \alpha_{X,g} = \alpha_{K,g} + \alpha_{L,g} = \sum_{r \in \mathcal{G}} \alpha_{g,r} = 1$ for every $g \in \mathcal{G}$.

Investment Network and Depreciation Because the investment baskets are Cobb-Douglas, investment expenditure shares will always be equal to $\vartheta_{g,r}$. Hence, we set the investment weights $\vartheta_{g,r}$ equal to the shares of investment purchases of industry g from r, as a share of total investment purchases by industry g, observed in the data of Vom Lehn and Winberry (2022) and Iovino et al. (2025). The sectoral depreciation rate δ_g is the depreciation rate of the sector's steady-state capital stock implied by its fixed-asset composition (taken from U.S. Bureau of Economic Analysis (2023a)) and the depreciation rates by asset type from Barro and Furman (2018): 8.8% for equipment, 2.0% for structures, 12.2% for research and development, and 19.6% for software and artistic originals. Concretely, we observe nominal investments by industry g in these four asset types a in U.S. Bureau of Economic Analysis (2023a), $I_{g,a}$, and with depreciation rates δ_a of asset type a, we set $\delta_g = \frac{\sum_a I_{g,a}}{\sum_a I_{g,a}/\delta_a}$.

C.3.2 Deterrence Block

Military Baskets Our data on military procurement consists of the purchases of goods and services by national defense in the 2017 BEA use table: the intermediate purchases of the national-defense government sector and defense investment in structures, equipment, and intellectual property. We first remove from these purchases the overhead services, such as facilities support, food service, and management consulting, which keep installations and personnel running but do not add to military power. The remaining 184 detailed industries are partitioned into the $J = 8$ baskets: land systems (military armored vehicles), air systems (aircraft, aircraft parts and engines), naval systems (ship and boat building), ammunition (ammunition, arms and ordnance; guided missiles; missile and space propulsion), structures, intellectual property (research and development, engineering, software), non-durables (fuel, pharmaceuticals, transportation, and other goods and services consumed within the year, which BEA expenses), and the residual other equipment. The within-basket weights $\psi_{j,g}$ and the basket shares θ_j are shares of these purchases in the 2017 BEA data (U.S. Bureau of Economic Analysis, 2023b). Spending on the baskets totals the postwar spending share $\theta = 3.37\%$ of GDP, the US average over 2015–2025 in the SIPRI data (Stockholm International Peace Research Institute, 2026), so by dropping the overhead we change the composition of modeled procurement but not its level. Table 8 lists every basket and its component sectors, and the overhead services excluded from the baskets.

Depreciation of Military Equipment The peacetime depreciation rates δ_j are the post-1970 means of current-cost depreciation over the net stock of Federal national-defense fixed assets in the BEA Fixed Assets tables (U.S. Bureau of Economic Analysis, 2025, Tables 7.1 and 7.3). For ammunition the peacetime depreciation rate is the FY2014–2019 mean of procurement (U.S. Department of Defense, 2015–2020) over the DoD Class V inventory of conventional ammunition and missiles (U.S. Department of Defense, 2014–2019, Table 1.7A), 12.7%. The wartime rates δ_j^w add combat losses. For land systems we use the Russia–Ukraine war: visually confirmed annual losses (destroyed, captured, and abandoned; Oryx, 2022a,b, as compiled by Drake, 2022) over the stock at the start of the year (*The Military Balance*, International Institute for Strategic Studies, 2022, 2023, 2024, 2025), 2022–2025, give attrition of 18% for Russia and 27% for Ukraine; added to the peacetime rate these give wartime rates of 34% and 43%, and we use their mean, $\delta^w = 38.3\%$. For air and naval systems we use the losses of the US armed forces in the Second World War, the only war in which US air and naval forces fought a peer adversary

at scale for several years.[23] For aircraft, attrition is the Army Air Forces' losses overseas from all causes, combat and operational, over the aircraft on hand at the start of the year (U.S. Army Air Forces, 1945, Tables 83 and 99), 24.2% per year on average over 1942–45; adding the peacetime rate gives $\delta^w = 34.3\%$. For ships, it is major combatants sunk or damaged beyond repair (battleships, carriers, cruisers, destroyers, destroyer escorts, and submarines; Naval History and Heritage Command, 2004) over the average number in commission during the year (Naval History and Heritage Command, 2017), 6.5% per year on average; adding the peacetime rate gives $\delta^w = 14.6\%$. Structures, other equipment, intellectual property, and non-durables keep their peacetime rates; ammunition is assumed to be expended within the year. Table 5 shows the depreciation rates.

Table 5: Depreciation of the military baskets in peace and in war.

Basket	Peacetime, δ_j	Combat attrition	Wartime, δ_j^w
Structures	0.020	0	0.020
Other equipment	0.116	0	0.116
Intellectual property	0.223	0	0.223
Non-durables	1.000	—	1.000
Land systems	0.155	0.228	0.383
Air systems	0.101	0.242	0.343
Naval systems	0.081	0.065	0.146
Ammunition	0.127	—	1.000

Duration of War In the Correlates of War data, combatants are approximately evenly matched in terms of military personnel at the beginning of inter-state wars. Hence, we set the probabilities π_T (the probability of a war having length T at parity of military power) equal to the frequencies of war lengths among the inter-state wars in the Correlates of War data (Sarkees and Wayman, 2010), 1816–2007, with wars of six years or more pooled in $T = 6$. The sensitivity κ is estimated on the same wars by a linear probability model, regressing an indicator for a war lasting more than one year on the initiator's share of the two sides' military personnel in the year before the war, from the Correlates of War National Material Capabilities data (Singer et al., 1972; Correlates of War Project, 2026). The initiator is the side coded as such in the dataset, and the other side comprises its opponents in the war's first year. Concretely, we run the following regression, where

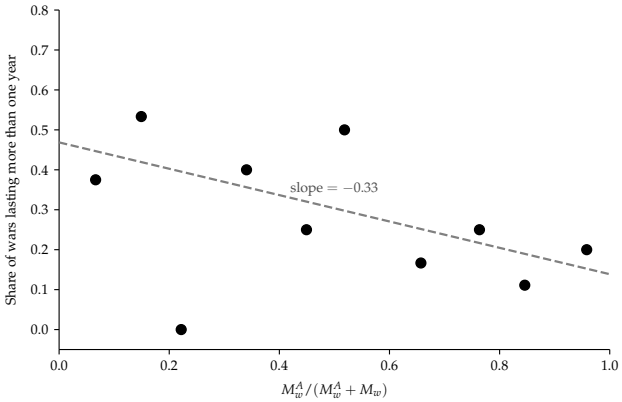
[23] To our knowledge, there is no comprehensive data on stocks of land systems for the US in the Second World War, which is why we did not calculate land system combat attrition rates for that conflict.

T_w denotes the length of war w in years, M_w^A and M_w are the military personnel of the initiator and the defender side, and ε_w is an error term:

$$\mathbb{1}\{T_w > 1\} = \alpha - \kappa \frac{M_w^A}{M_w^A + M_w} + \varepsilon_w$$

On the 87 wars with personnel data the estimated slope is $-\hat{\kappa} = -0.33$ (robust standard error 0.15). Figure 7 shows the relationship in the data between war length and the initiator's relative military personnel share.

Figure 7: War length and relative military personnel share of initiator.



Note: The figure shows the relationship between war length and the share of the initiator's military personnel out of total military personnel in Correlates of War data. The relative military personnel measure is binned by ten intervals of equal length.

Calibrating $\chi^{(T)}$ and ϕ The combat efficiency parameters $\chi_1^{(T)}, \ldots, \chi_{T-1}^{(T)}$ of a war of length T are pinned down by the sequence of the rise in military spending as a share of prewar GDP in major war booms, estimated by Marzian and Trebesch (2026). We denote this target sequence with σ^*, and it is equal to $\sigma^* = (5.3, 15, 19, 21.5, 20.5)\%$. We impose that in wars started from the no-industrial-policy peacetime steady state (Appendix C.4), in each year $s = 0, \ldots, T-2$, military spending exceeds its prewar level by σ_s^* of prewar GDP. Then, we back out the combat efficiency parameters $\chi^{(T)}$ that induce the same wartime procurement sequence.

Concretely, the combat efficiency parameters enter the wartime problem only through the procurement conditions (121)–(122). Fixing the path of military spending therefore fixes the wartime allocation independently of $\chi^{(T)}$, and the procurement conditions can be solved for the values of $\chi^{(T)}$ that make the path optimal: for any basket j procured in

positive quantities in period s, substituting (121) into (122) gives

$$\chi_{s+1}^{(T)} = \frac{1-\beta}{\beta^{T-s}} \frac{M_{j,s+1}}{\phi_j M_{s+1}} \Big[\frac{p_{m,j,s}}{C_s} - \beta(1-\delta_j^w) \frac{p_{m,j,s+1}}{C_{s+1}} \Big], \qquad s = 0, \ldots, T-3,$$

$$\chi_{T-1}^{(T)} = \frac{1-\beta}{\beta^2} \frac{M_{j,T-1}}{\phi_j M_{T-1}} \frac{p_{m,j,T-2}}{C_{T-2}}.$$

The parameter $\chi_0^{(T)}$ multiplies military power at the outbreak, M_0, which is predetermined when the war starts. It therefore affects no wartime decision, and the spending targets σ^* cannot pin it down. Instead, we set it so that the value of the arsenal at the outbreak lies on the trend of its values during the war. More specifically, let V_s denote the value of military power in period s: the value of increasing military power by a marginal unit, distributed across the baskets by their shares in the stocks, $w_{j,s} = M_{j,s} / \sum_k M_{k,s}$. A unit of stock j in period s is worth its contribution to the settlement in that period plus the value of the depreciated unit, which is in the position of a unit procured in period s. Using $\sum_j \phi_j = 1$, we get:

$$\begin{aligned} V_s &\equiv \sum_j w_{j,s} \Big[\frac{\beta^{T-s}}{1-\beta} \chi_s^{(T)} \phi_j \frac{M_s}{M_{j,s}} + (1-\delta_j^w)\, \mu_{j,s} \Big] \\ &= \frac{\beta^{T-s}}{1-\beta} \chi_s^{(T)} \frac{M_s}{\sum_k M_{k,s}} + \sum_j w_{j,s} (1-\delta_j^w)\, \mu_{j,s}, \qquad s = 0, \ldots, T-1. \end{aligned}$$

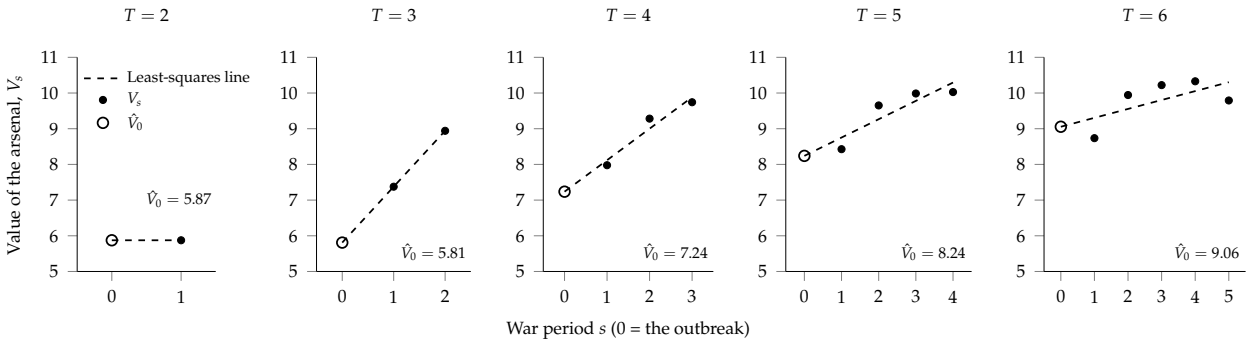
We fit a line to $V_1, \ldots, V_{T-1}$ by least squares and take its value $\hat{V}_0$ at $s = 0$; for $T = 2$, $\hat{V}_0 = V_1$. In our calibrated model, these values sit on an upward-sloping line, indicating that the value of a unit of military power is greater at points closer to the end of the war (see Figure 8). Setting $V_0 = \hat{V}_0$ and solving for $\chi_0^{(T)}$ gives

$$\chi_0^{(T)} = \frac{1-\beta}{\beta^T} \frac{\sum_k M_{k,0}}{M_0} \Big[\hat{V}_0 - \sum_j w_{j,0} (1-\delta_j^w)\, \mu_{j,0} \Big].$$

A one-year war has no periods after the outbreak, so we set $\chi_0^{(1)}$ to the mean of $\chi_0^{(T)}$ over $T = 2, \ldots, 6$.

Finally, the military-power weights ϕ_j are pinned down by the composition of procurement observed in the 2017 BEA use table. We set the weights such that the government that deters without industrial policy spends on basket j the spending share θ_j observed in the data. The combat efficiency parameters and the military-power weights depend on each other, since the wars that determine $\chi^{(T)}$ depend on ϕ and the procurement condi-

Figure 8: The value of the arsenal during the war and its extrapolation to the outbreak.



tions (124) depend on $\chi^{(T)}$ through the deterrence values. We therefore alternate between solving the procurement conditions for ϕ at given $\chi^{(T)}$ and inverting $\chi^{(T)}$ at given ϕ until convergence.[24] Table 6 reports the values of the calibrated $\chi^{(T)}$, and Table 7 shows the spending shares θ_j and the military-power weights ϕ.

Table 6: Combat efficiency parameters

T	$\chi_0^{(T)}$	$\chi_1^{(T)}$	$\chi_2^{(T)}$	$\chi_3^{(T)}$	$\chi_4^{(T)}$	$\chi_5^{(T)}$
1	0.417					
2	0.420	0.492				
3	0.222	0.204	0.639			
4	0.366	0.189	0.280	0.680		
5	0.482	0.202	0.258	0.308	0.697	
6	0.597	0.215	0.271	0.272	0.343	0.683

[24] Adversary's wartime military power, $\chi_0^{(T)}$, and η are recomputed at every step.

Table 7: Spending shares and military-power weights of the military baskets

Basket	Spending share, θ_j	Military-power weight, ϕ_j
Structures	0.030	0.023
Other equipment	0.232	0.173
Intellectual property	0.309	0.365
Non-durables	0.168	0.197
Land systems	0.007	0.007
Air systems	0.147	0.117
Naval systems	0.053	0.038
Ammunition	0.054	0.081

Adversary's Economy and the Level of Deterrence We assume that Adversary's economy mirrors Sovereign's: in a war of length T, started from the no-industrial-policy steady state, its military power, consumption, and labor supply follow Sovereign's own allocations, $M_s^A = M_s^{(T)}(\mathbf{K}^{ss}, \mathbf{M}^{ss})$ and likewise for $C_s^{A,(T)}$ and $\tilde{L}_s^{A,(T)}$. The economic costs $\mathcal{E}^{(T)}$ are evaluated over the T war periods and the postwar periods. Because the sequence of military powers is the same in the two economies, every settlement term of the deterrence function vanishes in the no-industrial-policy steady state. The deterrence function will be equal to Adversary's expected economic cost of war in that steady state. We set $\eta_t = \sum_T \pi_T \mathcal{E}^{(T)}$ so that the deterrence constraint binds at that steady state.

Table 8: The military baskets and their component sectors, and the overhead services excluded from the baskets

code	sector	code	sector
Structures (10 sectors)			
2334A0	Other residential structures	2332C0	Transportation structures & highways & streets
2332D0	Other nonresidential structures	233412	Multifamily residential structures
2332A0	Office & commercial structures	233230	Manufacturing structures
233262	Educational & vocational structures	233240	Power & communication structures
233210	Health care structures	233411	Single-family residential structures
Other equipment (105 sectors)			
334511	Search, detection, & navigation instr. mfg.	326220	Rubber & plastics hoses & belting mfg.
334220	Broadcast & wireless communications equip.	339990	All other miscellaneous mfg.
336320	Motor vehicle electrical & electronic equip. mfg.	332430	Metal can, box, & other metal container (light gauge) mfg.
334515	Electricity & signal testing instr. mfg.	336111	Automobile mfg.
334111	Electronic computer mfg.	315000	Apparel mfg.

continued on the next page

Table 8 (continued)

code	sector	code	sector
335313	Switchgear & switchboard apparatus mfg.	333994	Industrial process furnace & oven mfg.
336350	Motor vehicle transmission & power train parts mfg.	332720	Turned prod. & screw, nut, & bolt mfg.
332200	Cutlery & handtool mfg.	33399A	Other general purpose machinery mfg.
314900	Other textile prod. mills	333130	Mining & oil & gas field machinery mfg.
33441A	Other electronic component mfg.	336112	Light truck & utility vehicle mfg.
33291A	Valve & fittings other than plumbing	33329A	Other industrial machinery mfg.
326290	Other rubber prod. mfg.	335220	Major household appliance mfg.
335314	Relay & industrial control mfg.	33131B	Aluminum prod. mfg. from purchased aluminum
334210	Telephone apparatus mfg.	332320	Ornamental & architectural metal prod. mfg.
326210	Tire mfg.	3219A0	All other wood prod. mfg.
339113	Surgical appliance & supplies mfg.	334513	Industrial process variable instr. mfg.
336120	Heavy duty truck mfg.	336500	Railroad rolling stock mfg.
334510	Electromedical & electrotherapeutic apparatus mfg.	334610	Manufacturing & reproducing magnetic & optical media
332991	Ball & roller bearing mfg.	326110	Plastics packaging materials & unlaminated film & sheet mfg.
336390	Other motor vehicle parts mfg.	327999	Miscellaneous nonmetallic mineral prod.
333314	Optical instrument & lens mfg.	335991	Carbon & graphite prod. mfg.
33399B	Fluid power process machinery	331410	Nonferrous metal (exc. aluminum) smelting & refining
334118	Computer terminals & other computer peripheral equip. mfg.	332999	Other fabricated metal mfg.
335999	All other miscellaneous electrical equip. & component mfg.	334512	Automatic environmental control mfg.
336999	All other transportation equip. mfg.	3252A0	Synthetic rubber & artificial & synthetic fibers & filaments mfg.
333120	Construction machinery mfg.	333612	Speed changer, industrial high-speed drive, & gear mfg.
333914	Measuring, dispensing, & other pumping equip. mfg.	334514	Totalizing fluid meter & counting device mfg.
332500	Hardware mfg.	327200	Glass & glass prod. mfg.
333318	Other commercial & service industry machinery mfg.	335930	Wiring device mfg.
33451A	Watch, clock, & other measuring & controlling device mfg.	333613	Mechanical power transmission equip. mfg.
336310	Motor vehicle gasoline engine & engine parts mfg.	333316	Photographic & photocopying equip. mfg.
333611	Turbine & turbine generator set units mfg.	331420	Copper rolling, drawing, extruding & alloying
334112	Computer storage device mfg.	333517	Machine tool mfg.
334517	Irradiation apparatus mfg.	335210	Small electrical appliance mfg.
333415	Air conditioning, refrigeration, & warm air heating equip. mfg.	336991	Motorcycle, bicycle, & parts mfg.
316000	Leather & allied prod. mfg.	327910	Abrasive prod. mfg.
332119	Metal crown, closure, & other metal stamping (exc. automotive)	313100	Fiber, yarn, & thread mills
3363A0	Motor vehicle steering, suspension component (exc. spring), & brake systems mfg.	339940	Office supplies (exc. paper) mfg.
339112	Surgical & medical instrument mfg.	321910	Millwork
335312	Motor & generator mfg.	333514	Special tool, die, jig, & fixture mfg.

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Table 8 (continued)

code	sector	code	sector
314120	Curtain & linen mills	33721A	Office furniture & custom architectural woodwork & millwork mfg.
339114	Dental equip. & supplies mfg.	332996	Fabricated pipe & pipe fitting mfg.
333920	Material handling equip. mfg.	326190	Other plastics prod. mfg.
333618	Other engine equip. mfg.	321100	Sawmills & wood preservation
332310	Plate work & fabricated structural prod. mfg.	336360	Motor vehicle seating & interior trim mfg.
331313	Alumina refining & primary aluminum production	333413	Industrial & commercial fan & blower & air purification equip. mfg.
313200	Fabric mills	333991	Power-driven handtool mfg.
327991	Cut stone & stone prod. mfg.	333414	Heating equip. (exc. warm air furnaces) mfg.
327100	Clay prod. & refractory mfg.	335912	Primary battery mfg.
334290	Other communications equip. mfg.	325211	Plastics material & resin mfg.
335920	Communication & energy wire & cable mfg.	326150	Urethane & other foam prod. (exc. polystyrene) mfg.
336212	Truck trailer mfg.	326140	Polystyrene foam prod. mfg.
333912	Air & gas compressor mfg.		
Intellectual property (6 sectors)			
541700	Scientific research & development svcs.	541512	Computer systems design svcs.
541300	Architectural, engineering, & related svcs.	511200	Software publishers
541511	Custom computer programming svcs.	54151A	Other computer related svcs., including facilities management
Non-durables (54 sectors)			
324110	Petroleum refineries	311119	Other animal food mfg.
811200	Electronic & precision equip. repair & maintenance	482000	Rail transportation
325412	Pharmaceutical preparation mfg.	311910	Snack food mfg.
481000	Air transportation	322220	Paper bag & coated & treated paper mfg.
517210	Wireless telecommunications carriers (exc. satellite)	325413	In-vitro diagnostic substance mfg.
484000	Truck transportation	312110	Soft drink & ice mfg.
518200	Data processing, hosting, & related svcs.	322210	Paperboard container mfg.
31161A	Animal (exc. poultry) slaughtering, rendering, & processing	5111A0	Directory, mailing list, & other publishers
221100	Electric power generation, transmission, & distribution	311230	Breakfast cereal mfg.
221300	Water, sewage & other systems	311420	Fruit & vegetable canning, pickling, & drying
483000	Water transportation	324190	Other petroleum & coal prod. mfg.
311615	Poultry processing	312140	Distilleries
3259A0	All other chemical prod. & preparation mfg.	311940	Seasoning & dressing mfg.
517110	Wired telecommunications carriers	325610	Soap & cleaning compound mfg.
515100	Radio & television broadcasting	112A00	Animal production, exc. cattle & poultry & eggs
811300	Commercial & industrial machinery & equip. repair & maintenance	325310	Fertilizer mfg.
221200	Natural gas distribution	322230	Stationery prod. mfg.
325180	Other basic inorganic chemical mfg.	325620	Toilet preparation mfg.
323110	Printing	3118A0	Cookie, cracker, pasta, & tortilla mfg.
311513	Cheese mfg.	325320	Pesticide & other agricultural chemical mfg.
493000	Warehousing & storage	322120	Paper mills
311700	Seafood prod. preparation & packaging	325190	Other basic organic chemical mfg.

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Table 8 (continued)

code	sector	code	sector
311810	Bread & bakery prod. mfg.	532400	Commercial & industrial machinery & equip. rental & leasing
532100	Automotive equip. rental & leasing	322130	Paperboard mills
517A00	Satellite, telecommunications resellers, & all other telecommunications	311225	Fats & oils refining & blending
311300	Sugar & confectionery prod. mfg.	339920	Sporting & athletic goods mfg.
311990	All other food mfg.	322299	All other converted paper prod. mfg.
Land systems (1 sector)			
336992	Military armored vehicle, tank, & tank component mfg.		
Air systems (3 sectors)			
336411	Aircraft mfg.	336412	Aircraft engine & engine parts mfg.
336413	Other aircraft parts & auxiliary equip. mfg.		
Naval systems (2 sectors)			
336611	Ship building & repairing	336612	Boat building
Ammunition (3 sectors)			
336414	Guided missile & space vehicle mfg.	33641A	Propulsion units & parts for space vehicles & guided missiles
33299A	Ammunition, arms, ordnance, & accessories mfg.		
Overhead (23 sectors)			
230301	Nonresidential maintenance & repair	561700	Services to buildings & dwellings
561200	Facilities support svcs.	562000	Waste management & remediation svcs.
561300	Employment svcs.	5416A0	Environmental & other technical consulting svcs.
230302	Residential maintenance & repair	722A00	All other food & drinking places
721000	Accommodation	485000	Transit & ground passenger transportation
541800	Advertising, public relations, & related svcs.	512200	Sound recording industries
5191A0	News syndicates, libraries, archives & all other information svcs.	561400	Business support svcs.
722211	Limited-service restaurants	48A000	Scenic & sightseeing transportation & support activities for transportation
722110	Full-service restaurants	492000	Couriers & messengers
531ORE	Other real estate	541610	Management consulting svcs.
812300	Dry-cleaning & laundry svcs.	713900	Other amusement & recreation industries
811400	Personal & household goods repair & maintenance		

C.4 Sovereign's Peacetime Problem without Industrial Policy

We want to understand how using industrial policy to deter war can improve welfare. We therefore construct a benchmark problem in which Sovereign's government deters Adversary without using policy to target the allocation of capital. In this benchmark, the government accumulates military stocks $M_{j,t}$ to deter Adversary. It neither uses direct capital subsidies $s_{g,t}$ nor chooses procurement to influence the allocation of capital. Instead, it takes the sequence $\{\mathbf{K}_t\}_{t=0}^{\infty}$ as given when choosing military procurement. Under

these assumptions, the benchmark reduces to a simple public-good provision problem.

We solve this problem by deriving the government's optimal procurement choice. Notice that the value of a unit of procurement $m_{j,t}$, denoted $\varpi_{j,t}$, is equal to its present discounted deterrence value plus its depreciated value next period. Its marginal cost is $p_{m,j,t}$. At an interior solution, the government procures $m_{j,t}$ such that its marginal benefit, equal to its value $\varpi_{j,t}$, is set equal to its marginal cost. If the marginal benefit of $m_{j,t}$ at zero procurement is lower than its marginal cost, it will not buy any of it. Hence, denoting the shadow value of deterrence as γ_{t+1}, the first-order condition for procurement and the recursion for the shadow values are:

$$p_{m,j,t} \geq \varpi_{j,t}, \qquad m_{j,t} \geq 0, \qquad \text{with complementary slackness,} \tag{123}$$

$$\varpi_{j,t} = \beta \frac{C_t}{C_{t+1}} \left[\gamma_{t+1} \frac{\partial \mathcal{D}_{t+1}}{\partial M_{j,t+1}} + (1 - \delta_j)\, \varpi_{j,t+1} \right]. \tag{124}$$

In addition, it will accumulate stocks $M_{j,t}$ such that the deterrence constraint (34) is satisfied with equality:

$$\mathcal{D}_t(\mathbf{K}_t, \mathbf{M}_t) = \eta_t$$

C.5 Sovereign's Peacetime Problem

This section derives the optimality conditions of the peacetime planning problem (35) and compares them with the competitive equilibrium conditions in Appendix C.1 to prove Proposition 7.

The Lagrangian Attach to the date-t market-clearing condition of sector g the multiplier $\beta^t p_{g,t}/C_t$, to the law of motion of capital $\beta^t q_{g,t}/C_t$, to the law of motion of military stock j the multiplier $\beta^t \varpi_{j,t}/C_t$, and to the date-$(t+1)$ deterrence constraint $\beta^{t+1}\gamma_{t+1}/C_{t+1}$, where $1/C_t$ is the marginal utility of consumption. Normalizing the multipliers by the marginal utility of consumption in this way states them in units of the date-t consumption good. Substituting the aggregators for C_t, $\tilde{L}_t$, $V_{g,t}$, $X_{g,t}$, $Y_{g,t}$, $I_{g,t}$ and $m_{j,t}$, and with $I_{g,t} \geq 0$,

$m_{j,t} \geq 0$ and $\gamma_{t+1} \geq 0$, the Lagrangian is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & \log C_t - \frac{\tilde{L}_t^{1+1/\xi}}{1+1/\xi} + G_0 \\ & + \sum_{g \in \mathcal{G}} \frac{p_{g,t}}{C_t} \Big[Y_{g,t} - c_{g,t} - \sum_b x_{b,g,t} - \sum_b i_{b,g,t} - \sum_j m_{j,g,t} \Big] \\ & + \sum_{g \in \mathcal{G}} \frac{q_{g,t}}{C_t} \Big[(1-\delta_g) K_{g,t} + I_{g,t} - \frac{\varphi}{2} \Big(\frac{I_{g,t}}{K_{g,t}} - \delta_g \Big)^2 K_{g,t} - K_{g,t+1} \Big] \\ & + \sum_{j \in \mathcal{J}} \frac{\varpi_{j,t}}{C_t} \Big[(1-\delta_j) M_{j,t} + m_{j,t} - M_{j,t+1} \Big] \\ & + \beta \frac{\gamma_{t+1}}{C_{t+1}} \Big[\mathcal{D}_{t+1}(\mathbf{K}_{t+1}, \mathbf{M}_{t+1}) - \eta_{t+1} \Big] \Big\}.\end{aligned}$$

Static and Investment Conditions With the multipliers $p_{g,t}$ as the goods prices, $w_{g,t} \equiv p_{g,t} \partial Y_{g,t} / \partial L_{g,t}$ as the wages, and $q_{g,t}$ as the shadow value of capital, the planner's first-order conditions for consumption, labor, intermediate inputs, investment goods, and investment are the competitive equilibrium conditions for these choices in (109)–(114), so the planner's within-period allocation coincides with the competitive one. Two of them need a step. The optimality condition for consumption, the first below, inverts to the CES demand of (109); multiplying it by $c_{g,t}$ and summing over g, linear homogeneity of the aggregator gives $\sum_g p_{g,t} c_{g,t} = C_t$, so the consumption price index equals one. The condition for investment goods, the second below, is the demand of (113), and summing it over r gives the investment condition (114). The two conditions are

$$\Big(\epsilon_g \frac{C_t}{c_{g,t}} \Big)^{\frac{1}{\sigma_C}} = p_{g,t}, \qquad p_{r,t}\, i_{g,r,t} = q_{g,t} \Big[1 - \varphi \Big(\frac{I_{g,t}}{K_{g,t}} - \delta_g \Big) \Big] \vartheta_{g,r}\, I_{g,t}.$$

Capital The stock $K_{g,t+1}$ raises date-$(t+1)$ output, affects the date-$(t+1)$ deterrence constraint, and carries over into $K_{g,t+2}$. Only the date-$(t+1)$ deterrence constraint contains $K_{g,t+1}$, so a single multiplier, γ_{t+1}, appears. With $\partial K_{g,t+2} / \partial K_{g,t+1} = (1-\delta_g) + (\varphi/2)\big((I_{g,t+1}/K_{g,t+1})^2 - \delta_g^2 \big)$ implied by the law of motion, the first-order condition for $K_{g,t+1}$ is

$$\begin{aligned} q_{g,t} = \beta \frac{C_t}{C_{t+1}} \Big[& \gamma_{t+1} \frac{\partial \mathcal{D}_{t+1}}{\partial K_{g,t+1}} + p_{g,t+1}\, \mathrm{MPK}_{g,t+1} + (1-\delta_g)\, q_{g,t+1} \\ & + q_{g,t+1} \frac{\varphi}{2} \Big(\Big(\frac{I_{g,t+1}}{K_{g,t+1}} \Big)^2 - \delta_g^2 \Big) \Big]. \end{aligned} \tag{125}$$

Procurement and the Military Stocks Within each basket the planner buys goods at minimum cost, which gives the Cobb–Douglas demands of (117), so a unit of basket j costs $p_{m,j,t}$. The planner buys the basket up to the point where this cost equals the shadow value $\varpi_{j,t}$ of the stock, and buys none if the cost exceeds it. Military stocks enter neither production nor utility, so, unlike capital, they are valued for deterrence alone. With $\gamma_{t+1} \geq 0$, $\mathcal{D}_{t+1} \geq \eta_{t+1}$, and $\gamma_{t+1}\left(\mathcal{D}_{t+1} - \eta_{t+1}\right) = 0$, the conditions for procurement and for $M_{j,t+1}$ are

$$\begin{aligned} p_{m,j,t} \;&\geq\; \varpi_{j,t}, \qquad m_{j,t} \geq 0, \qquad \text{with complementary slackness,} \\ \varpi_{j,t} &= \beta \frac{C_t}{C_{t+1}} \left[\gamma_{t+1} \frac{\partial \mathcal{D}_{t+1}}{\partial M_{j,t+1}} + (1 - \delta_j)\, \varpi_{j,t+1} \right]. \end{aligned} \tag{126}$$

Proof of Proposition 7. Let the planner's allocation solve problem (35), with multipliers $p_{g,t}$, $q_{g,t}$, $\varpi_{j,t}$ and γ_t. Take $p_{g,t}$ as the goods prices, $w_{g,t} = p_{g,t}\partial Y_{g,t}/\partial L_{g,t}$ as the wages, and $1/(1+r_t) = \beta C_t / C_{t+1}$ as the bond price. Assume that the derivative $\frac{\partial \mathcal{D}_t}{\partial K_{g,t}}$ exists for all g, t. Let the government purchase the planner's $m_{j,g,t}$, balance its budget with the lump-sum tax of (118), and pay the capital subsidies

$$s_{g,t}\, P^I_{g,t} = \gamma_t \frac{\partial \mathcal{D}_t}{\partial K_{g,t}}, \qquad g \in \mathcal{G},\ t \geq 1. \tag{127}$$

By the static and investment conditions, the planner's allocation satisfies (109)–(114) at these prices, and its procurement, goods market clearing, and laws of motion satisfy (116)–(120). The two systems therefore differ in one condition only, the Euler equation for capital: where the competitive equation (115) has the subsidy $s_{g,t+1}P^I_{g,t+1}$, the planner's equation (125) has the deterrence value $\gamma_{t+1}\,\partial \mathcal{D}_{t+1}/\partial K_{g,t+1}$. Under (127) the two equations coincide, so the planner's allocation, together with the price system above, satisfies every condition of the competitive equilibrium of Appendix C.1; goods market clearing and the government budget then make the household's budget constraint hold by Walras' law. Solving (127) for the rate $s_{g,t}$ gives the statement of the proposition. □

C.6 Solution Method

This section describes the numerical method we use to solve Sovereign's peacetime planning problem (35), and calculate the optimal sectoral capital subsidies in Proposition 7.

Wars and the Deterrence Function First, we can solve for the wartime equilibrium using the perfect foresight method of Juillard (1996). We assume that the equilibrium allo-

cation reaches the postwar steady state in 8 periods—extending this to 30 has negligible effects on the quantitative results. Procurement follows (121)–(122) until the last period of the war and the spending rule from that period on. We set prices, consumption, and the shadow value of capital in the period after the last postwar period to their steady-state values, with the return on capital evaluated at the terminal capital stock. The allocation of military power across war lengths, together with Adversary's military power and its economic costs, and the war duration probabilities, give the deterrence function $\mathcal{D}(\mathbf{K}, \mathbf{M})$. Its derivatives with respect to the initial stocks follow from the implicit function theorem applied to the equilibrium conditions of each war.

The Optimal Deterrence Equilibrium Next, given the calibrated parameters and Adversary's behavior, the optimal steady state solves the peacetime first-order conditions of Appendix C.5 at constant allocations, stocks, and multipliers, displayed below without time subscripts and with C, $\tilde{L}$, Y_g, V_g, and X_g given by their aggregators. We solve these with a simple equation solver, and at each iteration of the algorithm, $\mathcal{D}$ and its derivatives are calculated using the method described in the previous paragraph. The subsidies are

then $s_g = \gamma\,(\partial \mathcal{D}/\partial K_g)/P_g^I$. The system of equations that we solve numerically is:

$$c_g = \epsilon_g\, p_g^{-\sigma_C} C, \qquad \Big[\sum_{g\in\mathcal{G}} \epsilon_g\, p_g^{1-\sigma_C}\Big]^{\frac{1}{1-\sigma_C}} = 1, \tag{128}$$

$$\frac{w_g}{C} = \tilde{L}^{\,1/\xi}\Big(\frac{L_g}{\tilde{L}}\Big)^{\frac{1}{\tau}}, \tag{129}$$

$$w_g = p_g\,\alpha_{V,g}\,\alpha_{L,g}\Big(\frac{Y_g}{V_g}\Big)^{\frac{1}{\sigma_Y}}\Big(\frac{V_g}{L_g}\Big)^{\frac{1}{\sigma_{VA}}}, \tag{130}$$

$$\mathrm{MPK}_g = \alpha_{V,g}\,\alpha_{K,g}\Big(\frac{Y_g}{V_g}\Big)^{\frac{1}{\sigma_Y}}\Big(\frac{V_g}{K_g}\Big)^{\frac{1}{\sigma_{VA}}}, \tag{131}$$

$$P_g^X = p_g\,\alpha_{X,g}\Big(\frac{Y_g}{X_g}\Big)^{\frac{1}{\sigma_Y}}, \tag{132}$$

$$x_{g,r} = \alpha_{g,r}^{\sigma_X}\Big(\frac{p_r}{P_g^X}\Big)^{-\sigma_X} X_g, \qquad P_g^X = \Big[\sum_{r\in\mathcal{G}} \alpha_{g,r}^{\sigma_X}\, p_r^{1-\sigma_X}\Big]^{\frac{1}{1-\sigma_X}}, \tag{133}$$

$$i_{g,r} = \vartheta_{g,r}\,\frac{P_g^I I_g}{p_r}, \qquad P_g^I = \prod_{r\in\mathcal{G}}\Big(\frac{p_r}{\vartheta_{g,r}}\Big)^{\vartheta_{g,r}}, \tag{134}$$

$$I_g = \delta_g K_g, \qquad q_g = P_g^I, \tag{135}$$

$$p_{m,j} = \prod_{g\in\mathcal{G}_j}\Big(\frac{p_g}{\psi_{j,g}}\Big)^{\psi_{j,g}}, \qquad m_{j,g} = \psi_{j,g}\,\frac{p_{m,j} m_j}{p_g}, \tag{136}$$

$$M_j = \frac{m_j}{\delta_j}, \tag{137}$$

$$Y_g = c_g + \sum_{b\in\mathcal{G}} x_{b,g} + \sum_{b\in\mathcal{G}} i_{b,g} + \sum_{j\in\mathcal{J}} m_{j,g}, \tag{138}$$

$$p_g\,\mathrm{MPK}_g + \gamma\,\frac{\partial \mathcal{D}}{\partial K_g} = \Big(\frac{1}{\beta} - 1 + \delta_g\Big) P_g^I, \quad g \in \mathcal{G}, \tag{139}$$

$$\gamma\,\frac{\partial \mathcal{D}}{\partial M_j} = \Big(\frac{1}{\beta} - 1 + \delta_j\Big) p_{m,j}, \quad j \in \mathcal{J}, \tag{140}$$

$$\mathcal{D}(\mathbf{K}, \mathbf{M}) = \eta. \tag{141}$$

C.7 Additional Quantitative Results

In this appendix, we present additional results related to the quantitative model.

C.7.1 Quantitative Results under Alternative Calibrations of χ_0

This section compares results under the baseline calibration of $\chi_0^{(T)}$ with two alternative calibrations. In the baseline, the outbreak combat efficiency $\chi_0^{(T)}$ is calibrated so that the

value of military power in the first period of the war lies on the trend of the value of military power during subsequent years of the war (Appendix C.3). Two alternative, extreme rules can be used: the minimum rule sets $\chi_0^{(T)}$ to the smallest of the inverted wartime parameters $\chi_1^{(T)}, \ldots, \chi_{T-1}^{(T)}$ of the same war, and the maximum rule to the largest; the one-year war takes the mean over $T \geq 2$, as in the baseline. Table 9 reports the aggregate outcomes. A higher χ_0 raises the marginal deterrence value of peacetime military power, which lowers the multiplier on the deterrence constraint and with it the subsidies and the gains from industrial policy; the ranking of the subsidies is nearly unchanged, with a rank correlation of 0.99 with the baseline under both rules (see Table 10 for the top 20 sectors under the three calibrations).

Table 9: Aggregate outcomes under alternative calibrations of the outbreak combat efficiency.

	baseline	minimum χ_0	maximum χ_0
$\mathcal{D}$	0.00	0.00	0.00
welfare	2.58	3.37	1.99
consumption	3.61	4.62	2.84
military procurement share	−28.20	−38.81	−20.85
real GDP	4.49	5.50	3.65
aggregate capital	9.01	11.33	7.20
labor supply	1.13	1.35	0.94

Note: All entries are percent deviations from the no-industrial-policy benchmark of the same calibration, as in Table 2, whose column the baseline repeats. The military procurement share is 3.37% in every benchmark. Welfare is expressed as the equivalent permanent increase in benchmark consumption; GDP and capital are Fisher quantity indices, and labor supply is the sum of sectoral labor.

Table 10: The twenty highest optimal capital subsidies under alternative calibrations of the outbreak combat efficiency

	baseline		minimum χ_0		maximum χ_0	
Sector	rank	s_g	rank	s_g	rank	s_g
Guided missile and space vehicle manufacturing	1	11.54	1	11.76	1	11.27
Propulsion units and parts for space vehicles and guided missiles	2	11.50	2	11.72	2	11.22
Ship building and repairing	3	8.93	3	9.35	4	7.83
Search, detection, and navigation instruments manufacturing	4	8.65	4	9.21	3	8.66
Military armored vehicle, tank, and tank component manufacturing	5	7.30	5	8.20	5	6.27
Ammunition, arms, ordnance, and accessories manufacturing	6	6.31	6	7.36	6	5.45
Other aircraft parts and auxiliary equipment manufacturing	7	6.14	8	6.78	7	5.31
Aircraft manufacturing	8	5.68	10	6.28	8	4.84
Truck transportation	9	5.65	7	6.86	9	4.76
Architectural, engineering, and related services	10	5.47	9	6.53	11	4.46
Electronic and precision equipment repair and maintenance	11	4.74	11	5.69	12	4.02
Electricity and signal testing instruments manufacturing	12	4.61	13	5.04	10	4.73
Other computer related services, including facilities management	13	4.06	12	5.05	15	3.24
Broadcast and wireless communications equipment	14	3.96	14	4.42	13	3.86
Water transportation	15	3.51	15	4.39	17	2.85
Switchgear and switchboard apparatus manufacturing	16	3.45	18	3.82	14	3.59
Computer systems design services	17	3.41	16	4.22	20	2.66
Air transportation	18	3.30	17	4.04	16	2.97
Aircraft engine and engine parts manufacturing	19	2.91	20	3.35	21	2.45
Other electronic component manufacturing	20	2.88	19	3.37	19	2.71
Motor vehicle electrical and electronic equipment manufacturing	21	2.77	22	3.12	18	2.76

Note: Subsidy rates s_g in percent, a rate on the replacement value of the sector's capital stock, and the sector's rank among the 370 sectors. Rows are the industries among the twenty highest subsidies in at least one of the three calibrations, in the baseline order.

C.7.2 The Full Ranking of Optimal Capital Subsidies

Table 11 reports the optimal capital subsidy of each of the 370 sectors. Sectors are ordered by decreasing subsidy, reading down the left column and then down the right column of each page.

Table 11: Optimal capital subsidies, all 370 sectors

rank	sector	s_g	rank	sector	s_g
1	Guided missile & space vehicle mfg.	11.54	24	Custom computer programming svcs.	2.20
2	Propulsion units & parts for space vehicles & guided missiles	11.50	25	Environmental & other technical consulting svcs.	2.16
3	Ship building & repairing	8.93	26	Scenic & sightseeing transportation & support activities for transportation	2.07
4	Search, detection, & navigation instr. mfg.	8.65	27	Wireless telecommunications carriers (except satellite)	2.06

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Table 11 (continued)

rank	sector	s_g	rank	sector	s_g
5	Military armored vehicle, tank, & tank component mfg.	7.30	28	Printed circuit assembly (electronic assembly) mfg.	2.05
6	Ammunition, arms, ordnance, & accessories mfg.	6.31	29	All other transportation equip. mfg.	2.04
7	Other aircraft parts & auxiliary equip. mfg.	6.14	30	All other chemical product & preparation mfg.	1.96
8	Aircraft mfg.	5.68	31	All other miscellaneous professional, scientific, & technical svcs.	1.95
9	Truck transportation	5.65	32	Semiconductor & related device mfg.	1.91
10	Architectural, engineering, & related svcs.	5.47	33	Management consulting svcs.	1.88
11	Electronic & precision equip. repair & maintenance	4.74	34	Manufacturing & reproducing magnetic & optical media	1.88
12	Electricity & signal testing instr. mfg.	4.61	35	Specialized design svcs.	1.86
13	Other computer related svcs., including facilities management	4.06	36	Carbon & graphite product mfg.	1.85
14	Broadcast & wireless communications equip.	3.96	37	Data processing, hosting, & related svcs.	1.82
15	Water transportation	3.51	38	Computer terminals & other computer peripheral equip. mfg.	1.75
16	Switchgear & switchboard apparatus mfg.	3.45	39	Office administrative svcs.	1.75
17	Computer systems design svcs.	3.41	40	Communication & energy wire & cable mfg.	1.73
18	Air transportation	3.30	41	Biological product (except diagnostic) mfg.	1.73
19	Aircraft engine & engine parts mfg.	2.91	42	Other support svcs.	1.73
20	Other electronic component mfg.	2.88	43	Ball & roller bearing mfg.	1.72
21	Motor vehicle electrical & electronic equip. mfg.	2.77	44	Satellite, telecommunications resellers, & all other telecommunications	1.71
22	Scientific research & development svcs.	2.71	45	Software publishers	1.70
23	Telephone apparatus mfg.	2.43	46	Relay & industrial control mfg.	1.68

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Table 11 (continued)

rank	sector	s_g	rank	sector	s_g
47	Other textile product mills	1.65	81	Plastics material & resin mfg.	1.30
48	Other basic inorganic chemical mfg.	1.64	82	Motor vehicle transmission & power train parts mfg.	1.29
49	Radio & television broadcasting	1.62	83	News syndicates, libraries, archives & all other information svcs.	1.28
50	Cutlery & handtool mfg.	1.60	84	Commercial & industrial machinery & equip. rental & leasing	1.27
51	Advertising, public relations, & related svcs.	1.58	85	Synthetic rubber & artificial & synthetic fibers & filaments mfg.	1.27
52	Computer storage device mfg.	1.57	86	Synthetic dye & pigment mfg.	1.26
53	Pharmaceutical preparation mfg.	1.57	87	Soap & cleaning compound mfg.	1.25
54	Employment svcs.	1.56	88	Facilities support svcs.	1.25
55	Other communications equip. mfg.	1.56	89	Management of companies & enterprises	1.25
56	Fluid power process machinery	1.54	90	Valve & fittings other than plumbing	1.24
57	Watch, clock, & other measuring & controlling device mfg.	1.52	91	Pesticide & other agricultural chemical mfg.	1.24
58	Automatic environmental control mfg.	1.51	92	Motor & generator mfg.	1.23
59	Couriers & messengers	1.49	93	Fertilizer mfg.	1.20
60	Accounting, tax preparation, bookkeeping, & payroll svcs.	1.47	94	Wiring device mfg.	1.19
61	Medicinal & botanical mfg.	1.47	95	Electronic computer mfg.	1.19
62	Petrochemical mfg.	1.46	96	All other forging, stamping, & sintering	1.19
63	Industrial gas mfg.	1.46	97	Commercial & industrial machinery & equip. repair & maintenance	1.17
64	Printing ink mfg.	1.45	98	Office supplies (except paper) mfg.	1.16
65	Rail transportation	1.42	99	Newspaper publishers	1.15
66	Motorcycle, bicycle, & parts mfg.	1.41	100	Paint & coating mfg.	1.15
67	Wired telecommunications carriers	1.41	101	Mechanical power transmission equip. mfg.	1.15
68	Business support svcs.	1.41	102	Other motor vehicle parts mfg.	1.14
69	Internet publishing & broadcasting & Web search portals	1.41	103	Leather & allied product mfg.	1.14
70	Other basic organic chemical mfg.	1.41	104	Lessors of nonfinancial intangible assets	1.14
71	In-vitro diagnostic substance mfg.	1.40	105	Petroleum & petroleum prod.	1.14
72	Heavy duty truck mfg.	1.39	106	Totalizing fluid meter & counting device mfg.	1.14
73	Petroleum refineries	1.39	107	Toilet preparation mfg.	1.13
74	Boat building	1.38	108	Audio & video equip. mfg.	1.13
75	Investigation & security svcs.	1.38	109	Spring & wire product mfg.	1.12
76	Other rubber product mfg.	1.37	110	Travel arrangement & reservation svcs.	1.12
77	Printing	1.34	111	Adhesive mfg.	1.11
78	Directory, mailing list, & other publishers	1.33	112	Apparel mfg.	1.11
79	Photographic svcs.	1.32	113	Optical instrument & lens mfg.	1.07
80	Periodical Publishers	1.32	114	Hardware mfg.	1.06

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Table 11 (continued)

rank	sector	s_g	rank	sector	s_g
115	Electric lamp bulb & part mfg.	1.06	149	Drugs & druggists' sundries	0.88
116	All other miscellaneous mfg.	1.06	150	Services to buildings & dwellings	0.88
117	Other support activities for mining	1.06	151	Plastics pipe, pipe fitting, & unlaminated profile shape mfg.	0.88
118	Speed changer, industrial high-speed drive, & gear mfg.	1.06	152	Support activities for printing	0.87
119	Legal svcs.	1.04	153	Lime & gypsum product mfg.	0.87
120	All other miscellaneous electrical equip. & component mfg.	1.04	154	Motor home mfg.	0.87
121	Turbine & turbine generator set units mfg.	1.04	155	Tire mfg.	0.86
122	Primary battery mfg.	1.04	156	Laminated plastics plate, sheet (except packaging), & shape mfg.	0.86
123	Storage battery mfg.	1.03	157	Book publishers	0.86
124	Concrete pipe, brick, & block mfg.	1.03	158	Travel trailer & camper mfg.	0.85
125	Copper rolling, drawing, extruding & alloying	1.03	159	Ophthalmic goods mfg.	0.84
126	Major household appliance mfg.	1.03	160	Dental laboratories	0.84
127	Motor vehicle steering, suspension component (except spring), & brake systems mfg.	1.02	161	Other animal food mfg.	0.83
128	Alumina refining & primary aluminum production	1.02	162	Nonferrous metal (except aluminum) smelting & refining	0.82
129	Coating, engraving, heat treating & allied activities	1.02	163	Glass & glass product mfg.	0.82
130	Veterinary svcs.	1.02	164	Other plastics product mfg.	0.81
131	Water, sewage & other systems	1.01	165	Paper bag & coated & treated paper mfg.	0.81
132	Doll, toy, & game mfg.	1.01	166	Oil & gas extraction	0.79
133	General & consumer goods rental	1.00	167	Textile & fabric finishing & fabric coating mills	0.79
134	Surgical appliance & supplies mfg.	1.00	168	Turned product & screw, nut, & bolt mfg.	0.79
135	Jewelry & silverware mfg.	0.99	169	Polystyrene foam product mfg.	0.79
136	Seafood product preparation & packaging	0.98	170	Custom roll forming	0.78
137	Small electrical appliance mfg.	0.98	171	Curtain & linen mills	0.75
138	Plate work & fabricated structural product mfg.	0.97	172	Motor vehicle gasoline engine & engine parts mfg.	0.75
139	Cheese mfg.	0.96	173	Warehousing & storage	0.75
140	Other petroleum & coal prod. mfg.	0.95	174	Wholesale electronic markets & agents & brokers	0.75
141	Other engine equip. mfg.	0.95	175	Industrial process variable instr. mfg.	0.75
142	Poultry processing	0.93	176	Aluminum product mfg. from purchased aluminum	0.75
143	Nonferrous metal (except copper & aluminum) rolling, drawing, extruding & alloying	0.92	177	Paperboard mills	0.74
144	Household appliances & electrical & electronic goods	0.92	178	Cut stone & stone product mfg.	0.74
145	Metal crown, closure, & other metal stamping (except automotive)	0.90	179	Personal & household goods repair & maintenance	0.74
146	Sporting & athletic goods mfg.	0.89	180	Other nondurable goods merchant wholesalers	0.74
147	Machine shops	0.89	181	Fiber, yarn, & thread mills	0.73
148	Animal (except poultry) slaughtering, rendering, & processing	0.89	182	Dry-cleaning & laundry svcs.	0.73

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Table 11 (continued)

rank	sector	s_g	rank	sector	s_g
183	Abrasive product mfg.	0.72	217	All other converted paper product mfg.	0.61
184	Paperboard container mfg.	0.72	218	Fabricated pipe & pipe fitting mfg.	0.61
185	Dental equip. & supplies mfg.	0.72	219	Spectator sports	0.60
186	Ground or treated mineral & earth mfg.	0.71	220	Flavoring syrup & concentrate mfg.	0.60
187	Stationery product mfg.	0.71	221	Tobacco mfg.	0.60
188	Beef cattle ranching & farming, including feedlots & dual-purpose ranching & farming	0.70	222	Poultry & egg production	0.60
189	Grocery & related product wholesalers	0.70	223	Urethane & other foam product (except polystyrene) mfg.	0.60
190	Metal can, box, & other metal container (light gauge) mfg.	0.69	224	Electric power generation, transmission, & distribution	0.60
191	Ornamental & architectural metal prod. mfg.	0.69	225	Soft drink & ice mfg.	0.60
192	Ferrous metal foundries	0.69	226	Mineral wool mfg.	0.59
193	Fabric mills	0.68	227	Religious organizations	0.59
194	Plastics packaging materials & unlaminated film & sheet mfg.	0.68	228	All other food mfg.	0.59
195	Steel product mfg. from purchased steel	0.68	229	Personal care svcs.	0.58
196	Civic, social, professional, & similar organizations	0.67	230	Truck trailer mfg.	0.58
197	Other personal svcs.	0.67	231	Pulp mills	0.58
198	Other furniture related product mfg.	0.66	232	Grantmaking, giving, & social advocacy organizations	0.58
199	Motor vehicle & motor vehicle parts & supplies	0.66	233	Bread & bakery product mfg.	0.58
200	Plastics bottle mfg.	0.66	234	Heating equip. (except warm air furnaces) mfg.	0.58
201	Automotive equip. rental & leasing	0.66	235	Breweries	0.57
202	Distilleries	0.65	236	Sugar & confectionery product mfg.	0.57
203	Clay product & refractory mfg.	0.65	237	Snack food mfg.	0.57
204	Other durable goods merchant wholesalers	0.64	238	Natural gas distribution	0.57
205	Iron & steel mills & ferroalloy mfg.	0.64	239	Animal production, except cattle & poultry & eggs	0.56
206	Nonferrous metal foundries	0.64	240	Photographic & photocopying equip. mfg.	0.56
207	Breakfast cereal mfg.	0.64	241	Waste management & remediation svcs.	0.56
208	Copper, nickel, lead, & zinc mining	0.63	242	All other food & drinking places	0.56
209	Paper mills	0.63	243	Other household nonupholstered furniture	0.55
210	Asphalt paving mixture & block mfg.	0.62	244	Independent artists, writers, & performers	0.55
211	Automotive repair & maintenance (including car washes)	0.62	245	Professional & commercial equip. & supplies	0.55
212	Other fabricated metal mfg.	0.62	246	Soybean & other oilseed processing	0.54
213	Death care svcs.	0.61	247	Wet corn milling	0.54
214	Carpet & rug mills	0.61	248	Flour milling & malt mfg.	0.54
215	Wineries	0.61	249	Fats & oils refining & blending	0.54
216	Nonstore retailers	0.61	250	Offices of dentists	0.53

continued on the next page

Table 11 (continued)

rank	sector	s_g	rank	sector	s_g
251	Rubber & plastics hoses & belting mfg.	0.53	285	Grain farming	0.42
252	Offices of other health practitioners	0.53	286	Limited-service restaurants	0.41
253	Seasoning & dressing mfg.	0.53	287	Residential maintenance & repair	0.41
254	Museums, historical sites, zoos, & parks	0.52	288	Electromedical & electrotherapeutic apparatus mfg.	0.40
255	Promoters of performing arts & sports & agents for public figures	0.52	289	Coal mining	0.38
256	Nonupholstered wood household furniture mfg.	0.52	290	Other real estate	0.37
257	Cement mfg.	0.52	291	All other wood product mfg.	0.36
258	Upholstered household furniture mfg.	0.51	292	Plumbing fixture fitting & trim mfg.	0.36
259	Fruit & vegetable canning, pickling, & drying	0.51	293	Support activities for agriculture & forestry	0.36
260	Sanitary paper product mfg.	0.51	294	Analytical laboratory instrument mfg.	0.35
261	Outpatient care centers	0.51	295	Other crop farming	0.35
262	Dry, condensed, & evaporated dairy product mfg.	0.51	296	Iron, gold, silver, & other metal ore mining	0.33
263	Home health care svcs.	0.51	297	Sound recording industries	0.33
264	Offices of physicians	0.50	298	Child day care svcs.	0.31
265	Pipeline transportation	0.50	299	Individual & family svcs.	0.31
266	Nonresidential maintenance & repair	0.50	300	Multifamily residential structures	0.31
267	Elementary & secondary schools	0.50	301	Power-driven handtool mfg.	0.30
268	Coffee & tea mfg.	0.49	302	Community food, housing, & other relief svcs., including rehabilitation svcs.	0.30
269	Medical & diagnostic laboratories	0.49	303	Greenhouse, nursery, & floriculture production	0.30
270	Cookie, cracker, pasta, & tortilla mfg.	0.49	304	Millwork	0.30
271	Fishing, hunting & trapping	0.49	305	Single-family residential structures	0.29
272	Fluid milk & butter mfg.	0.49	306	Miscellaneous nonmetallic mineral prod.	0.29
273	Dog & cat food mfg.	0.49	307	Vegetable & melon farming	0.28
274	Power, distribution, & specialty transformer mfg.	0.49	308	Oilseed farming	0.28
275	Frozen food mfg.	0.48	309	Fruit & tree nut farming	0.28
276	Junior colleges, colleges, universities, & professional schools	0.48	310	Irradiation apparatus mfg.	0.27
277	Ice cream & frozen dessert mfg.	0.48	311	Accommodation	0.26
278	Other ambulatory health care svcs.	0.48	312	Hospitals	0.25
279	Full-service restaurants	0.47	313	Amusement parks & arcades	0.24
280	Performing arts companies	0.47	314	Automobile mfg.	0.24
281	Dairy cattle & milk production	0.46	315	Other amusement & recreation industries	0.24
282	Other educational svcs.	0.45	316	Gambling industries (except casino hotels)	0.24
283	Transit & ground passenger transportation	0.44	317	Forestry & logging	0.23
284	Lighting fixture mfg.	0.43	318	Other residential structures	0.23

continued on the next page

Table 11 (continued)

rank	sector	s_g	rank	sector	s_g
319	Sawmills & wood preservation	0.21	345	Veneer, plywood, & engineered wood product mfg.	-0.24
320	Machinery, equip., & supplies	0.20	346	Stone mining & quarrying	-0.24
321	Surgical & medical instrument mfg.	0.19	347	Industrial mold mfg.	-0.29
322	Industrial & commercial fan & blower & air purification equip. mfg.	0.18	348	Railroad rolling stock mfg.	-0.29
323	Cutting & machine tool accessory, rolling mill, & other metalworking machinery mfg.	0.18	349	Other general purpose machinery mfg.	-0.37
324	Other concrete product mfg.	0.17	350	Showcase, partition, shelving, & locker mfg.	-0.38
325	Residential mental health, substance abuse, & other residential care facilities	0.17	351	Drilling oil & gas wells	-0.41
326	Ready-mix concrete mfg.	0.17	352	Special tool, die, jig, & fixture mfg.	-0.42
327	Nursing & community care facilities	0.16	353	Lawn & garden equip. mfg.	-0.42
328	Motion picture & video industries	0.15	354	Cable & other subscription programming	-0.45
329	Asphalt shingle & coating materials mfg.	0.15	355	Machine tool mfg.	-0.47
330	Construction machinery mfg.	0.12	356	Material handling equip. mfg.	-0.49
331	Other nonmetallic mineral mining & quarrying	0.10	357	Packaging machinery mfg.	-0.54
332	Motor vehicle metal stamping	0.10	358	Other industrial machinery mfg.	-0.56
333	Air conditioning, refrigeration, & warm air heating equip. mfg.	0.09	359	Power boiler & heat exchanger mfg.	-0.56
334	Motor vehicle seating & interior trim mfg.	0.05	360	Metal tank (heavy gauge) mfg.	-0.58
335	Transportation structures & highways & streets	0.02	361	Power & communication structures	-0.59
336	Other commercial & service industry machinery mfg.	0.02	362	Air & gas compressor mfg.	-0.61
337	Measuring, dispensing, & other pumping equip. mfg.	-0.00	363	Farm machinery & equip. mfg.	-0.63
338	Semiconductor machinery mfg.	-0.04	364	Other nonresidential structures	-0.64
339	Mining & oil & gas field machinery mfg.	-0.07	365	Manufacturing structures	-0.68
340	Motor vehicle body mfg.	-0.07	366	Institutional furniture mfg.	-0.91
341	Educational & vocational structures	-0.09	367	Office furniture & custom architectural woodwork & millwork mfg.	-0.99
342	Industrial process furnace & oven mfg.	-0.12	368	Office & commercial structures	-1.01
343	Wood kitchen cabinet & countertop mfg.	-0.13	369	Health care structures	-1.04
344	Light truck & utility vehicle mfg.	-0.19	370	Sign mfg.	-1.07

C.7.3 Military Centrality

In this section, we look at how the "centrality" of a sector to military demand affects the optimal subsidy that the government should pay for its production capacity. To do so, we construct a "military centrality" measure which quantifies how important a given sector's output is to producing military power.

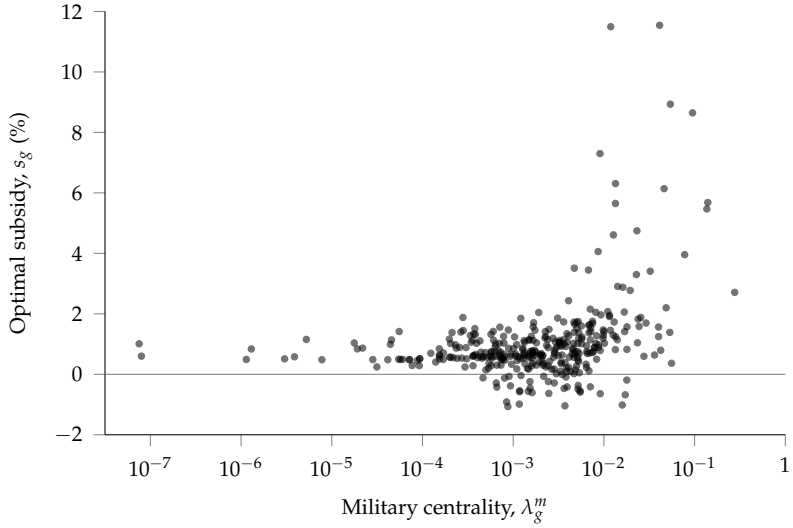
To simplify the analysis, let us assume that all sectors pay a fixed share of their costs on individual intermediate inputs in order to produce a dollar of output. We will focus on steady states, and we assume that sectors pay a fixed share of their costs on individual investment goods to cover depreciation. Let $\psi_g = \sum_j \theta_j \psi_{j,g}$ be sector g's share of defense procurement; we assume that when the government expands military power, it spends a fixed share of its expenditures on each sector's output. A dollar of revenue of sector

g generates demand for its suppliers through two channels. First, it needs $a_{X,g}\,\omega_{g,r}$ dollars of supplier r's output as intermediate inputs, where $a_{X,g}$ is the intermediate share of sector g's costs and $\omega_{g,r}$ the share of supplier r in its intermediate purchases. Second, it needs $\beta\delta_g/(1-\beta+\beta\delta_g)\,a_{K,g}\,\vartheta_{g,r}$ dollars as investment goods, where $a_{K,g}$ is the capital share of its costs and the fraction $\beta\delta_g/(1-\beta+\beta\delta_g)$ is the part of the user cost of capital that replaces depreciation. We take all shares from the 2017 BEA data (U.S. Bureau of Economic Analysis, 2023b), the investment network shares ϑ from Vom Lehn and Winberry (2022) and Iovino et al. (2025), and the depreciation rates δ_g of Appendix C.3. We define the vector of sectoral military centralities as:

$$\boldsymbol{\lambda}^m = \left(\mathbf{I} - A'\right)^{-1}\boldsymbol{\psi}, \qquad \text{where} \qquad A_{g,r} = a_{X,g}\,\omega_{g,r} + \frac{\beta\delta_g}{1-\beta+\beta\delta_g}\,a_{K,g}\,\vartheta_{g,r}.$$

We interpret this as a rough measure of each sector's importance to expanding military power. Figure 9 plots the optimal subsidies against it.

Figure 9: Optimal capital subsidies and military centrality.



Note: Each dot is one of the 370 sectors; the 18 sectors with no exposure to defense demand ($\lambda_g^m = 0$) are omitted. The horizontal line marks a zero subsidy. Military centrality is defined in Appendix C.7.3.

C.7.4 Capacity Exercises

The goal of the exercises reported in this section is to understand how industrial capacity in the twenty sectors with the highest optimal subsidies influences deterrence. In these

exercises, we start the six-year war from the no-industrial-policy steady state and raise the capital stock of one of the twenty sectors of Table 3 to its level in the optimal peacetime steady state at the outbreak, one sector at a time, holding the combat efficiency parameters $\chi^{(T)}$ fixed; the wartime government re-optimizes its procurement. All reported changes are relative to the same war without the capital increase. Table 12 reports the change in the price of each military basket, averaged over the five war years in which the government procures. Table 13 reports the increase in the sector's capital stock, the change in the sector's total output over the six war years and the average change in its price, and splits the rise in the quantity it sells across consumption, investment, intermediate, and military buyers.

Table 12: Change in military basket prices

sector (capital raised to its optimum)	str	eq	int	nd	land	air	naval	ammo
Guided missile & space vehicle mfg.	-0.006	+0.015	+0.006	+0.008	+0.019	+0.016	+0.018	-0.457
Propulsion units & parts for space vehicles & guided missiles	-0.022	+0.060	+0.022	+0.029	+0.066	+0.053	+0.063	-1.533
Ship building & repairing	-0.025	+0.078	+0.022	+0.020	+0.062	+0.052	-2.207	+0.048
Search, detection, & navigation instr. mfg.	-0.175	-2.724	+0.134	+0.203	+0.463	+0.427	-0.207	+0.276
Military armored vehicle, tank, & tank component mfg.	-0.014	+0.026	-0.011	+0.020	-6.873	+0.012	+0.045	+0.038
Ammunition, arms, ordnance, & accessories mfg.	-0.012	+0.056	+0.035	+0.042	+0.067	+0.074	+0.068	-1.515
Other aircraft parts & auxiliary equip. mfg.	-0.013	+0.114	+0.030	+0.040	+0.077	-1.113	+0.121	+0.079
Aircraft mfg.	-0.063	+0.504	+0.125	+0.042	+0.167	-4.427	+0.472	+0.333
Truck transportation	-0.008	+0.029	+0.026	-0.371	+0.049	+0.054	+0.042	+0.038
Architectural, engineering, & related svcs.	-0.143	+0.474	-1.475	+0.286	+0.733	+0.620	+0.525	+0.535
Electronic & precision equip. repair & maintenance	-0.049	+0.073	+0.033	-0.684	+0.100	+0.103	+0.088	+0.080
Electricity & signal testing instr. mfg.	-0.003	-0.144	+0.005	+0.017	+0.019	+0.031	+0.028	+0.024
Other computer related svcs., including facilities management	+0.002	+0.015	-0.035	+0.008	+0.025	+0.019	+0.019	+0.016
Broadcast & wireless communications equip.	+0.002	-0.583	+0.045	-0.008	+0.140	+0.081	+0.224	+0.005
Water transportation	+0.002	+0.008	+0.003	-0.034	+0.018	+0.015	-0.081	+0.011
Switchgear & switchboard apparatus mfg.	-0.006	-0.043	+0.001	+0.006	+0.006	+0.011	+0.011	+0.008
Computer systems design svcs.	+0.026	+0.052	-0.101	+0.024	+0.063	+0.056	+0.058	+0.039
Air transportation	+0.034	+0.210	+0.183	-0.549	+0.294	-1.316	+0.296	+0.258
Aircraft engine & engine parts mfg.	+0.008	+0.028	+0.010	+0.015	+0.013	-0.160	+0.032	+0.026
Other electronic component mfg.	+0.004	-0.037	-0.003	+0.004	+0.014	+0.017	+0.018	+0.017

Note: Percent change in each basket's price, mean over the five war years in which the government procures; in red, the largest fall in each row. str: structures; eq: other equipment; int: intellectual property; nd: non-durables.

Table 13: Capital, output, price, and the composition of the rise in sales

	% change			composition of the rise in sales (%)			
sector (capital raised to its optimum)	capital	output	price	cons.	inv.	interm.	military
Guided missile & space vehicle mfg.	+303.70	+0.45	-0.52	1.4	1.4	0.8	96.4
Propulsion units & parts for space vehicles & guided missiles	+294.67	+2.43	-9.39	7.0	0.0	46.1	47.0
Ship building & repairing	+71.93	+4.49	-1.96	0.4	2.8	0.1	96.7
Search, detection, & navigation instr. mfg.	+61.96	+10.65	-7.79	3.1	8.0	3.6	85.3
Military armored vehicle, tank, & tank component mfg.	+56.32	+4.47	-5.83	5.4	0.0	2.3	92.3
Ammunition, arms, ordnance, & accessories mfg.	+55.08	+4.52	-5.76	28.2	0.0	6.2	65.6
Other aircraft parts & auxiliary equip. mfg.	+36.85	+2.71	-5.31	28.1	0.0	26.2	45.7
Aircraft mfg.	+40.17	+6.91	-4.90	8.2	17.5	1.6	72.7
Truck transportation	+33.70	+2.68	-4.63	26.9	0.0	1.5	71.6
Architectural, engineering, & related svcs.	+36.36	+3.75	-3.76	6.1	9.8	4.1	80.0
Electronic & precision equip. repair & maintenance	+37.78	+2.59	-4.70	14.9	0.0	0.6	84.5
Electricity & signal testing instr. mfg.	+24.63	+2.75	-4.29	19.3	19.6	7.1	54.0
Other computer related svcs., including facilities management	+21.16	+0.71	-1.92	32.3	0.0	1.4	66.3
Broadcast & wireless communications equip.	+22.85	+3.14	-3.25	16.5	26.6	9.8	47.1
Water transportation	+23.64	+1.18	-1.69	70.4	0.0	0.4	29.2
Switchgear & switchboard apparatus mfg.	+19.73	+1.64	-2.32	11.5	29.0	1.3	58.1
Computer systems design svcs.	+19.28	+0.96	-1.55	23.3	27.5	1.7	47.5
Air transportation	+35.79	+5.34	-7.00	66.0	0.0	0.8	33.2
Aircraft engine & engine parts mfg.	+19.46	+1.73	-3.24	70.6	0.4	19.9	9.1
Other electronic component mfg.	+17.24	+0.29	-1.55	50.3	0.0	11.1	38.6

Note: Capital: percent increase in the sector's capital stock from the no-industrial-policy steady state to the optimal peacetime steady state. Output: percent change in the sector's total output over the six war years. Price: percent change in the sector's price, mean over the six war years. The last four columns split the rise in the quantity sold across buyers and sum to one hundred.

C.7.5 Industrial Priorities across Conflict Scenarios

Table 14 reports the optimal capital subsidies of the twenty industries in Table 3 in the land-, air-, and naval-war scenarios of Section 5.4.

Table 14: Optimal capital subsidies of the twenty baseline priorities across conflict scenarios.

Sector	Land	Air	Naval
Guided missile and space vehicle manufacturing	11.50	11.47	11.58
Propulsion units and parts for space vehicles and guided missiles	11.40	11.40	11.51
Ship building and repairing	0.81	1.11	8.98
Search, detection, and navigation instruments manufacturing	8.66	8.13	9.06
Military armored vehicle, tank, and tank component manufacturing	7.09	3.15	2.80
Ammunition, arms, ordnance, and accessories manufacturing	6.26	6.18	6.45
Other aircraft parts and auxiliary equipment manufacturing	0.99	6.07	1.02
Aircraft manufacturing	1.09	5.61	1.14
Truck transportation	5.75	5.59	5.87
Architectural, engineering, and related services	5.67	5.42	5.80
Electronic and precision equipment repair and maintenance	4.87	4.72	4.95
Electricity and signal testing instruments manufacturing	4.95	4.63	5.04
Other computer related services, including facilities management	4.37	4.14	4.33
Broadcast and wireless communications equipment	4.16	4.11	4.27
Water transportation	3.15	2.98	3.88
Switchgear and switchboard apparatus manufacturing	3.81	3.51	3.83
Computer systems design services	3.64	3.44	3.66
Air transportation	2.51	3.88	2.39
Aircraft engine and engine parts manufacturing	1.07	3.00	1.07
Other electronic component manufacturing	3.09	2.89	3.10

Note: Subsidy rates s_g in percent, a rate on the replacement value of the sector's capital stock. Rows are the twenty industries with the highest subsidies in the baseline (Table 3), in the baseline order.

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